

Homework 4

Deadline: 31th July (23:55 JST), 2022

You can submit your solution at NUCT, by email (henrik.bachmann@math.nagoya-u.ac.jp) or hand it in physically.

Exercise 14. For $k \geq 1$ show that $\zeta_{\mathcal{A}}(k) = 0$ by just using the equations (*) and (\sqcup) in Conjecture 3.3

Exercise 15. Prove Lemma 4.1., i.e. show that for any index $\mathbf{k} = (k_1, \dots, k_r)$ the polynomial

$$R_{\mathbf{k}}(X; T) = \sum_{j=0}^r (-1)^{k_1 + \dots + k_j} \zeta^*(k_j, k_{j-1}, \dots, k_1; T + X) \zeta^*(k_{j+1}, \dots, k_r; T - X)$$

does not depend on T .

Exercise 16. Prove the linear shuffle relations for multiple harmonic q -series at roots of unity (Theorem 4.9), i.e. show that for $n \geq 2$ and any primitive n -th root of unity ζ_n and $w, v \in \widehat{\mathfrak{H}}^1$ we have

$$H_{n-1}(w \sqcup_q v; \zeta_n) = (-1)^{\text{wt}(w)} H_{n-1}(\psi(w)v; \zeta_n).$$