



POLYA'S **FOUR** STEP PROBLEM SOLVING METHOD

Henrik Bachmann

Graduate School of Mathematics
Nagoya University



NU Nagoya
University
Mirai Education Project

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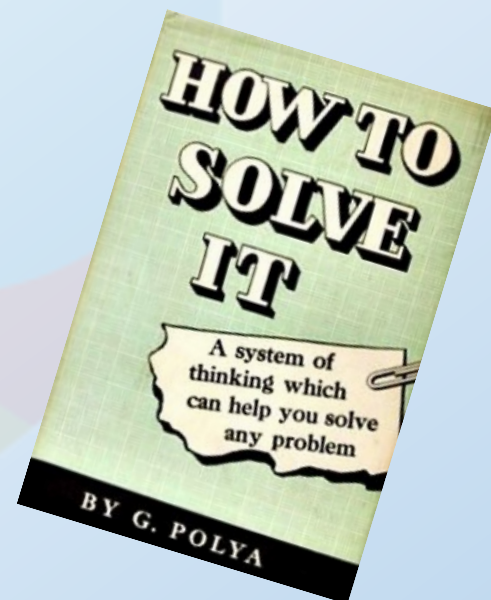
WHO IS POLYA?

George Pólya

1887-1985



- Was a teacher and mathematician.
- Published a book in 1945: “**How To Solve It**”, explaining how to become a better problem solver.



WHAT IS A “PROBLEM”?

- A **problem** is a situation, condition, person, or thing that needs attention and needs to be dealt with or solved.
- Not solving a problem leads to undesirable consequences.

“The solution to any problem lies in asking the right questions”

Polya’s four step method: A systematic way to answer/attack questions

POLYA'S **FOUR** STEPS

Polya's strategy to answer questions is given by the following four steps:

- 1 Understand the question**
- 2 Make a plan**
- 3 Carry out the plan**
- 4 Look back & Review**

1

Understand the question

This seems so obvious that it is often ignored!

Ask yourself the following questions

- ❖ What are you asked to find or show?
- ❖ Can you restate the question in your own words?
- ❖ What part/information of the question is (un)important?
- ❖ Can you think of a picture or a diagram that might help you understand the question?
- ❖ Is there enough information to enable you to find a solution?
- ❖ Do you understand all the words used in stating the question?
- ❖ Do you need to ask a question to get the answer?
- ❖ Why might this problem be difficult/easy?

2

Make a plan

You must start somewhere so try something. How are you going to attack the question?

Possible Strategies

- ❖ Draw pictures or diagrams.
- ❖ Eliminate possibilities.
- ❖ Be systematic.
- ❖ Solve a simpler version of the problem & Consider special cases.
- ❖ Guess and check. Trial and error. Guess and test.
- ❖ Look for a pattern or patterns.
- ❖ Make a list / Write down keywords.



3

Carry out the plan

This is the step where you carry out the steps of your plan.

Answering the question

- ❖ Try to use the strategy chosen in step 2.
- ❖ If this strategy does not work, try another one.



4

Look back & Review

Finally, in this last step you look back reviewing and checking your results.

Ask yourself the following questions

- ❖ Did you answer the question? Is your result reasonable?
- ❖ What would change if you change the question a bit?
- ❖ Is there a better/more interesting version of the question?
- ❖ Is there another way of doing the problem which may be simpler?
- ❖ Can the question or method be generalized to be useful for future problems?

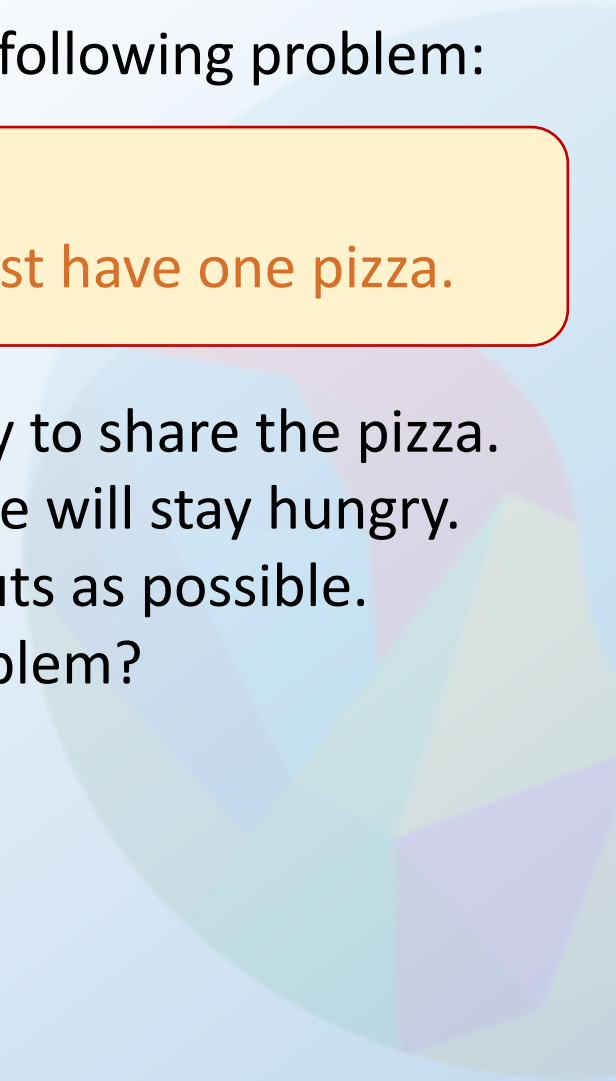


EXAMPLE: Sharing a pizza

Let us try to use Polya's method to solve the following problem:

Problem:

You are at a party with 11 people and you just have one pizza.

- This is a problem since you need to find a way to share the pizza.
 - Consequences if you do not share it: Someone will stay hungry.
 - Assume you want to cut the pizza with few cuts as possible.
 - What are possible questions to solve the problem?
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- Assume you want to cut the pizza with few cuts as possible.
- What are possible questions to solve the problem?

Question:

Can you cut a pizza into 11 pieces with just 4 cuts?

1 Understand the question

Pizza problem

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Ask yourself the following questions

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- ❖ **What part/information of the question is (un)important?**
- ❖ Can you think of a picture or a diagram that might help you understand the problem?
- ❖ Is there enough information to enable you to find a solution?
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- ❖ Why might this problem be difficult/easy?

1 Understand the question

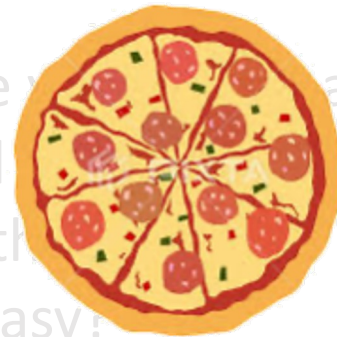
Pizza problem

Question:

Can you cut a pizza into 11 pieces with just 4 cuts?

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- ❖ Can you restate the problem in your own words?
- ❖ What part/information of the question is (un)important?
- ❖ Can you think of a picture or a diagram that might help you understand the problem?
- ❖ Is there enough information to enable you to find a solution?
- ❖ Do you understand all the words used in the problem?
- ❖ Do you need to ask a question to get the information you need?
- ❖ Why might this problem be difficult/easy?



1 Understand the question

Pizza problem

Question:

Can you cut a pizza into 11 pieces with just 4 cuts?

What kind of knife? Straight cuts? All pieces have some crust?
Pieces of same size? Can we stack pizza pieces?
Does it always need to go through the center?

- ❖ Is there enough information to enable you to find a solution?
- ❖ Do you understand all the words used in stating the problem?
- ❖ Do you need to ask a question to get the answer?
- ❖ Why might this problem be difficult/easy?

1 Understand the question

Pizza problem

Question:

Can you cut a pizza into 11 pieces with just 4 cuts?

If it is possible: Easy, since we can maybe give an explicit way of cutting the pizza.

If it is not possible: Maybe difficult because we need to argue why it is not possible.

- ❖ Is there a solution?
- ❖ Do you understand all the terms used in stating the problem?
- ❖ Do you need to ask a question to get the answer?
- ❖ Why might this problem be difficult/easy?

Question:

Can you cut a pizza into 11 pieces with just 4 cuts?

Possible Strategies

- ❖ Draw pictures or diagrams.
- ❖ Eliminate possibilities
- ❖ Be systematic.
- ❖ Solve a simpler version of the question & Consider special cases
- ❖ Guess and check. Trial and error. Guess and test.
- ❖ Look for a pattern or patterns.
- ❖ Make a list / Write down keywords.

Question:

Can you cut a pizza into 11 pieces with just 4 cuts?

Possible Strategies

- ❖ Draw pictures or diagrams.
- ❖ Eliminate possibilities
- ❖ Be systematic. (If it is not possible)
- ❖ Solve a simpler version of the problem & Consider special cases
- ❖ Guess and check. Trial and error. Guess and test.
- ❖ Look for a pattern or patterns.
- ❖ Make a list / Write down keywords.

3

Carry out the plan

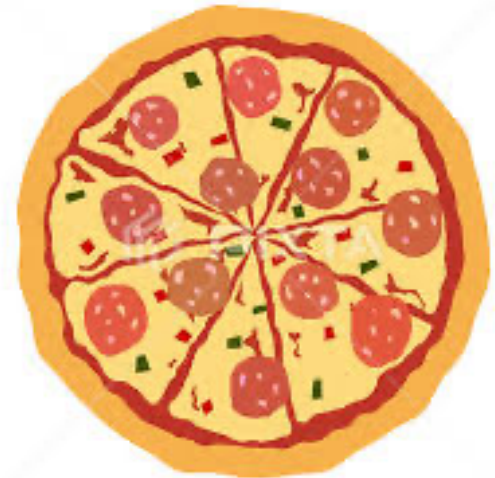
Pizza problem

Question:

Can you cut a pizza into 11 pieces with just 4 cuts?

Strategies:

- ❖ Draw pictures or diagrams.
- ❖ Trial and error.



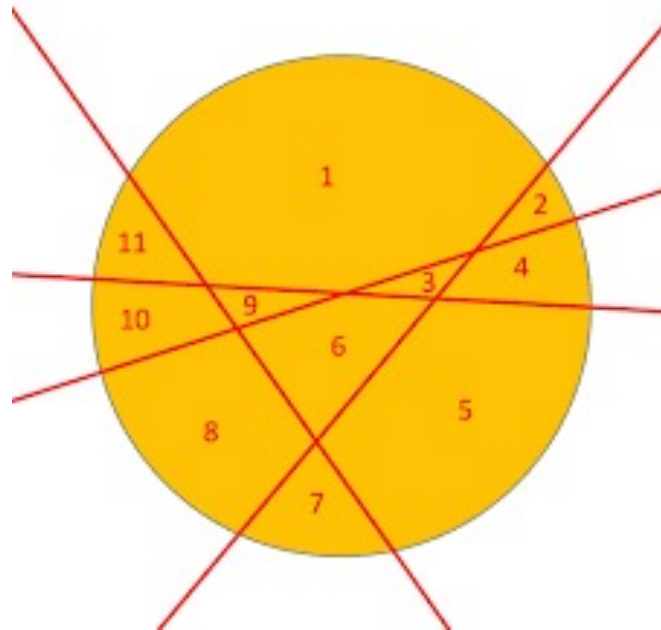
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Carry out the plan

Pizza problem

Question:

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Question:

Can you cut a pizza into 11 pieces with just 4 cuts?

Ask yourself the following questions

- ❖ Did you answer the question? Is your result reasonable?
- ❖ What would change if you change the question a bit?
- ❖ Is there a better/more interesting version of the question?
- ❖ Is there another way of doing the problem which may be simpler?
- ❖ Can the problem or method be generalized to be useful for future problems?

4

Look back & Review

Pizza problem

Question:

Can you cut a pizza into 11 pieces with just 4 cuts?

Ask yourself the following questions

- ❖ Did you answer the question? Is your result reasonable?
- ❖ What would change if you change the question a bit?
- ❖ Is there a better/more... can?
- ❖ Is there... pler?
- ❖ Can... ture problems?

Yes! We gave an explicit way of cutting a pizza into 11 pieces with 4 cuts.

Question:

Can you cut a pizza into 11 pieces with just 4 cuts?

Ask yourself the following questions

- ❖ Did you answer the question? Is your result reasonable?
- ❖ What would change if you change the question a bit?
- ❖ Is there a better/more interesting version of the question?
- ❖ Is there another way of doing the problem, with a different question?
- ❖ Can the problem be generalized to other problems?

For example: If we restrict to “4 cuts which all go through the center” then there is no way to get 11 pieces.

Question:

Can you cut a pizza into 11 pieces with just 4 cuts?

Ask yourself the following questions

- ❖ Did you answer the question? Is your result reasonable?
- ❖ What would change if you change the question a bit?
- ❖ Is there a better/more interesting version of the question?
- ❖ Is there another way of doing this?
- ❖ Can the question be made more challenging?

What is the maximum number of pieces you can obtain with 4 cuts?

What is the maximum number of (same sized) pieces (with crust) you can obtain with n cuts (which go through the center)?

What shape of knife would give more pieces?

Replace pizza (circle) by another shape.

Question:

Can you cut a pizza into 11 pieces with just 4 cuts?

Ask yourself the following questions

- ❖ Did you answer the question? Is your result reasonable?
- ❖ What would change if you change the question a bit?
- ❖ Is there a better/more interesting version of the question?
- ❖ Is there another way of doing the problem which may be simpler?
- ❖ Can the problem or method be generalized to be useful for future problems?

Pizza is not important...

- Of course there are more serious problems to solve.
- Most of them are not mathematical, i.e. maybe do not have only one “correct” solution.

Global catastrophic risks [edit]

Main articles: [Global catastrophic risk](#) and [Potential sources of global catastrophic risk](#)

Not all of these risks are independent, because the majority, if not all of them are a result of human activity.

- [Biodiversity loss](#)
- [Climate change](#)
- [Destructive artificial intelligence](#)
- [Environmental disaster](#)
- [Nuclear holocaust](#)
- [Pandemic](#)
 - current example: [COVID-19 pandemic](#)
- [Biotechnology risk](#)
- [Molecular nanotechnology](#)

Polya's method for general questions

- For general questions, especially Step 1 is important!

Question:

Is it good to ...

How to ...

invest in solar energy stop overpopulation
stop eating meat control the development of AI
do more sports decrease taxes ... ?
fight covid-19

1

Ask yourself the following questions

- ❖ What are you asked to find or show?
- ❖ Can you restate the question in your own words?
- ❖ What part/information of the question is (un)important?
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... ?

1

Ask yourself the following questions

- ❖ What are you asked to find or show?

There might not be one single answer.

Maybe one can just give pro/contra arguments.

- ❖ Can you restate the problem?
- ❖ What is the question?
- ❖ Do you understand the problem?
- ❖ Why might this problem be difficult/easy?

Polya's method for general questions

- For general question, especially Step 1 is important!

Question:

Is it good to ...

How to ...

invest in solar energy stop overpopulation
stop eating meat control the development of AI
do more sports decrease taxes
fight covid-19

... ?

1

What does "good" mean? What would be "bad"?

Good for who? Economy? Humans? Animals? Me?

- ❖ Is there enough information to enable you to find a solution?
- ❖ **Do you understand all the words used in stating the problem?**
- ❖ Do you need to ask a question to get the answer?
- ❖ Why might this problem be difficult/easy?

Polya's method for general questions

Question:

Is it good to ...
How to ...

invest in solar energy stop overpopulation
stop eating meat control the development of AI
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fight covid-19

2

Possible Strategies

- ❖ Draw pictures or diagrams.
- ❖ Eliminate possibilities
- ❖ Be systematic.
- ❖ Solve a simpler version of the problem & Consider special cases
- ❖ Guess and check. Trial and error. Guess and test.
- ❖ Look for a pattern or patterns.
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Polya's method for general questions

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How to ...

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... ?

2

Possible Strategies

- ❖ Draw pictures or diagrams.
- ❖ Eliminate possibilities
- ❖ Be systematic.
- ❖ Solve a simpler version of the problem & Consider special cases
- ❖ Guess and check
- ❖ Look for a pattern

Just focus on one aspect. For example, just consider the affects on economy and forget any other aspects (ethics etc.).

Polya's method for general questions

Question:

Is it good to ...
How to ...

invest in solar energy stop overpopulation
stop eating meat control the development of AI
do more sports decrease taxes ... ?
fight covid-19

2

Possible Strategies

- ❖ Draw pictures or diagrams.
- ❖ Eliminate possibilities
- ❖ Be systematic
- ❖ Make a mindmap or pro/contra list of arguments.
- ❖ Look for a pattern or patterns
- ❖ Make a list / Write down keywords.

Review

1

Understand the problem

2

Make a plan

3

Carry out the plan

4

Look back & Review

- **Try to think about problems and come up with related questions.**
 - **How do the steps 1,2,3 and 4 look like for these questions?**

Exercise

Let's discuss together the possible steps of Polya's method to answer the following questions

"How can you make your teacher happy in school?"

"Two numbers differ by 3 and have a product of 10.
What are the two numbers?"

Steps of Polya's method

- 1 Understand the problem
- 2 Make a plan
- 3 Carry out the plan
- 4 Look back & Review

Assignment: Problems of “new technology”

In the assignment you should think about problems related to **new technology** (of now or the future).

new technology

(some examples)

Image & speech recognition

Artificial intelligence

Self driving cars ...

Virtual reality

.... and so on

TikTok

Social media

LINE

Instagram ...

Time travel

Quantum computing

Scientific breakthroughs

... CRISPR

- You are not restricted to the above topics. Choose whatever you are interested in!
- When introducing yourself to your group members give one example of “new technology” you are personally interested in.

Assignment



Applying New Technologies to Global Issues

20th January 2024

- 1) Decide together with your group members on a global issue or social problem that interests all of you.
- 2) Think of a new technology (for example some kind of AI) that would be useful in solving the problem that interests you and describe its positive aspects.

(This „new technology“ does not need to exist now. If you want, you can be creative and come up with some future technology)

- 3) Formulate problems that might occur due to the use of the new technology you choose in 2).

“The solution to any problem lies in asking the right questions”

- 4) Come up with a question related to the problem(s) in 3) and answer it by using Polya’s four step problem solving method.

Create a presentation where you present your results of 1) - 4) .



20th January 2024

Recall Polya's four step problem solving method:

1

Understand the question

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- ❖ Why might this question be difficult/easy?

2

Make a plan

- ❖ Draw pictures or diagrams.
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Carry out the plan

- ❖ Try to use the strategy chosen in step 2.
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- ❖ Did you answer the question? Is your result reasonable?
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