Linear Algebra I

Fall Semester 2023

G30 Program, Nagoya University

Please take a copy of Homework 1 and Course information
(if you did not take one in the tutorial)
Both can also be found on the homepage:
https://www.henrikbachmann.com/la1_2023.html

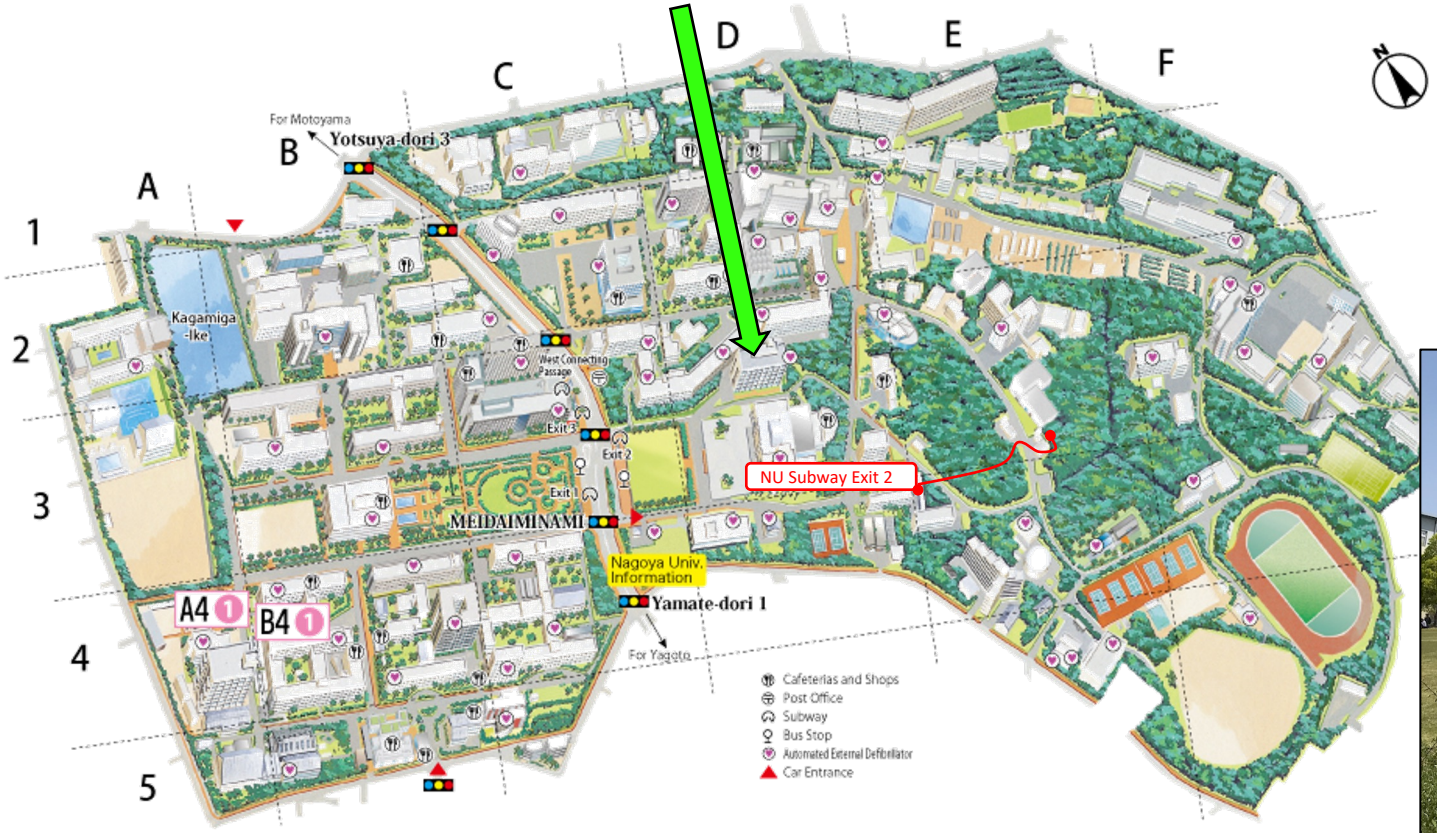
Who is teaching?

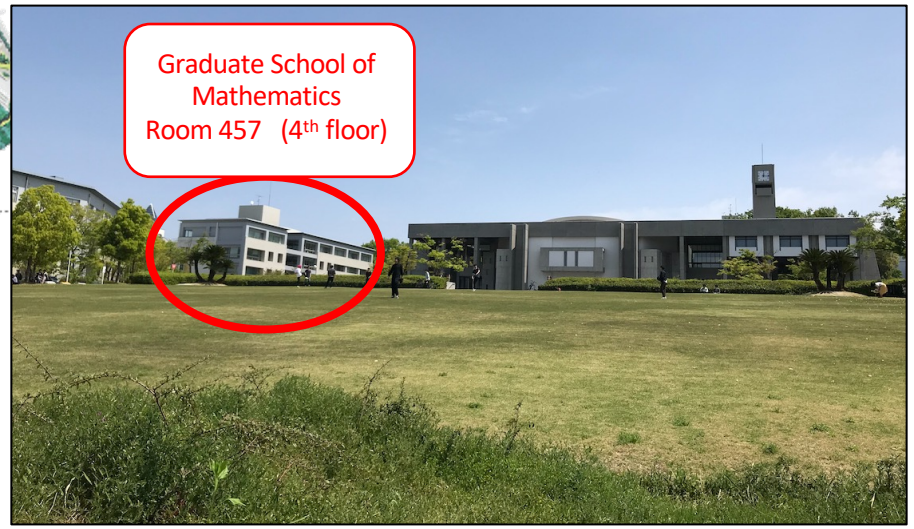
Henrik Bachmann

www.henrikbachmann.com
henrik.bachmann@math.nagoya-u.ac.jp

From here

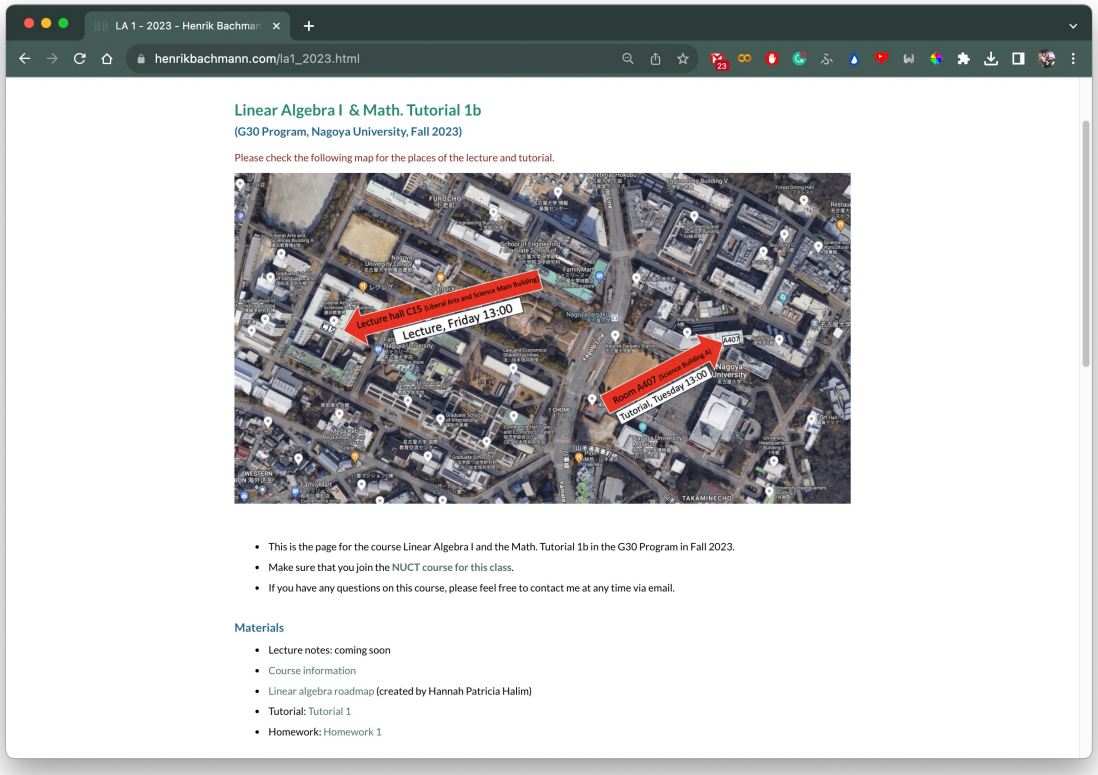
Has his office here





What where ?

Most materials can be found here: https://www.henrikbachmann.com/la1_2023.html

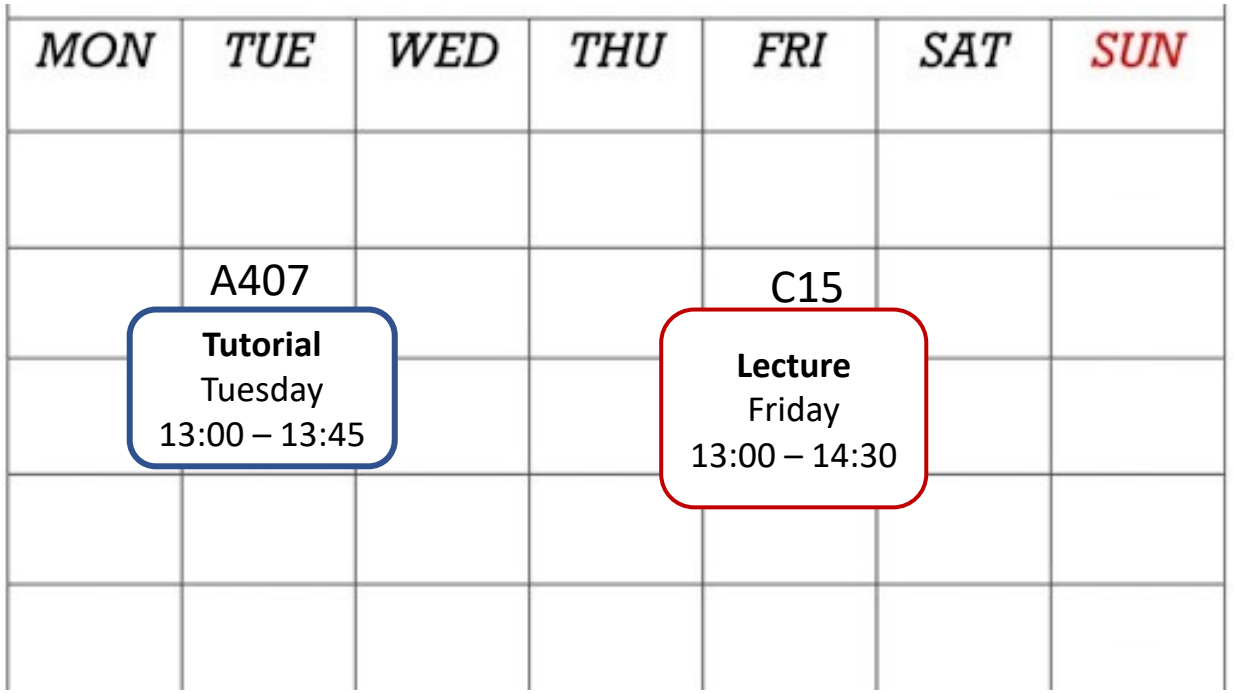


- **Lecture notes:** Contains the content of the lecture and additional examples/remarks/proofs.
- **Handwritten notes:** The notes I use for the lecture
- **Tutorial notes:** The notes we create during the tutorial
- **Homework:** The homework assignments. Need to be submitted in NUCT.

- **Lecture notes (2020):** Handwritten lecture notes from 2020
- **Lecture videos (2020):** Recorded lectures from 2020

I encourage everyone to take their own notes

When do we meet? What to do?



Deadline for homework
(Sunday 23:55)

What you should do:

- Join the Lectures **and / or** watch the recordings of the Lectures
- If you just watch the videos of 2020 please also check the lecture notes of this year.
- Attend the Tutorial each Tuesday 13:00-13:40.
  There we will discuss the content of the previous week and the homework.
- Submit Homework at TACT (around every two weeks)

If you have any questions I am always open for office hours (Zoom or in my office)

Grading & Exams

Your grade for the course “Linear Algebra I” and “Mathematics Tutorial 1b” will be the same. This grade will be calculated as follows:

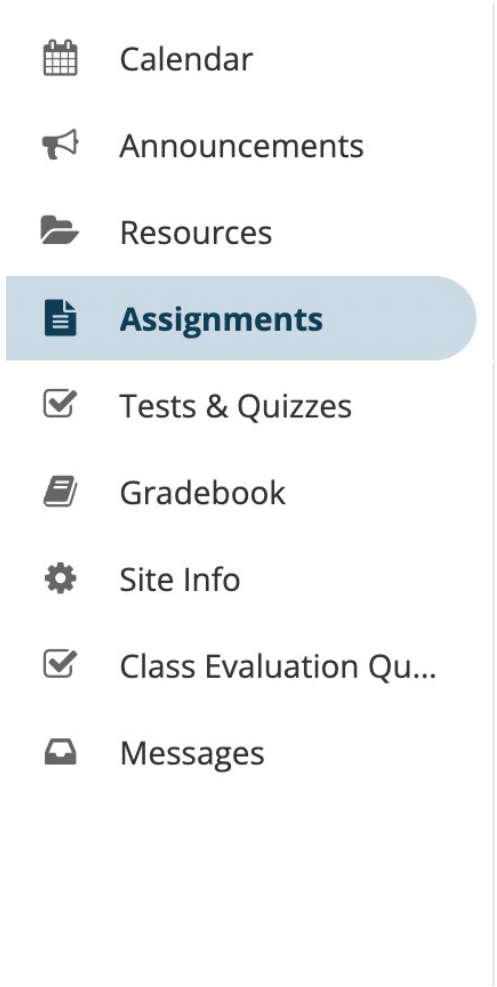
Homework:	30%	Roughly every second week 2-3 Exercises
Midterm exam:	30%	17th November 2023 in the lecture
Final exam:	40%	2nd February 2024 in the lecture

The grading scale will be A+, A, B, C, C-, F.

For those just taking Linear Algebra I:

You can choose to also do the homework and use the same grading above.
Or you can choose to just write the midterm (40%) and final exam (60%).

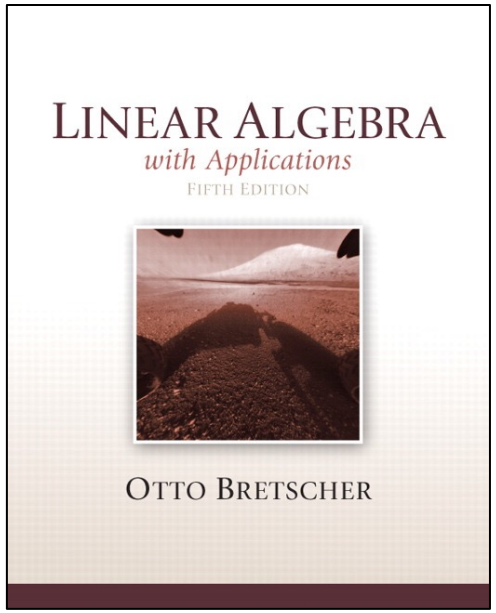
Homework

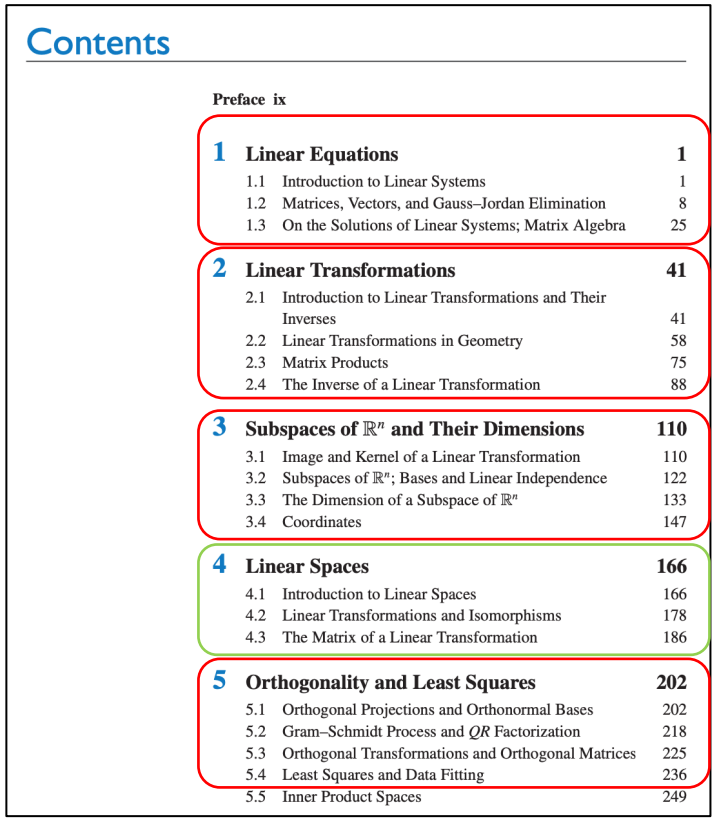


- Write the homework solutions down by hand (paper, tablet) or by computer (Latex; contact me if you want to learn it!).
- Create one pdf file which contains your solution. For this you can, for example, use a scanner app on your phone.
- Use the following format as a filename: "*Familyname_Givenname_LA1_HWX.pdf*", where X = Homework number.
- Please make sure that the solutions are readable and do not need to be turned by 90 degrees.
- Submit the Homework in TACT. Resubmissions are always possible.

Other references?

There are tons of sources out there for Linear Algebra 1.
This course is based on the book “Linear Algebra with Applications” by Otto Bretscher.





Contents

Preface ix

1 Linear Equations 1
1.1 Introduction to Linear Systems 1
1.2 Matrices, Vectors, and Gauss–Jordan Elimination 8
1.3 On the Solutions of Linear Systems; Matrix Algebra 25

2 Linear Transformations 41
2.1 Introduction to Linear Transformations and Their Inverses 41
2.2 Linear Transformations in Geometry 58
2.3 Matrix Products 75
2.4 The Inverse of a Linear Transformation 88

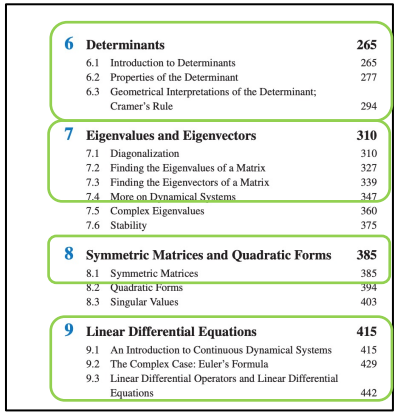
3 Subspaces of $\mathbb{R}^n$ and Their Dimensions 110
3.1 Image and Kernel of a Linear Transformation 110
3.2 Subspaces of $\mathbb{R}^n$; Bases and Linear Independence 122
3.3 The Dimension of a Subspace of $\mathbb{R}^n$ 133
3.4 Coordinates 147

4 Linear Spaces 166
4.1 Introduction to Linear Spaces 166
4.2 Linear Transformations and Isomorphisms 178
4.3 The Matrix of a Linear Transformation 186

5 Orthogonality and Least Squares 202
5.1 Orthogonal Projections and Orthonormal Bases 202
5.2 Gram–Schmidt Process and QR Factorization 218
5.3 Orthogonal Transformations and Orthogonal Matrices 225
5.4 Least Squares and Data Fitting 236
5.5 Inner Product Spaces 249

Linear Algebra I

Linear Algebra II



6 Determinants 265
6.1 Introduction to Determinants 265
6.2 Properties of the Determinant 277
6.3 Geometrical Interpretations of the Determinant; Cramer's Rule 294

7 Eigenvalues and Eigenvectors 310
7.1 Diagonalization 310
7.2 Finding the Eigenvalues of a Matrix 327
7.3 Finding the Eigenvectors of a Matrix 339
7.4 More on Dynamical Systems 347
7.5 Complex Eigenvalues 360
7.6 Stability 375

8 Symmetric Matrices and Quadratic Forms 385
8.1 Symmetric Matrices 385
8.2 Quadratic Forms 394
8.3 Singular Values 403

9 Linear Differential Equations 415
9.1 An Introduction to Continuous Dynamical Systems 415
9.2 The Complex Case: Euler's Formula 429
9.3 Linear Differential Operators and Linear Differential Equations 442

YouTube:

- 3Blue1Brown Series on Linear Algebra https://youtu.be/fNk_zzaMoSs
- Khan Academy https://youtu.be/xyAuNHPsq-g
- My YouTube channel https://youtu.be/r23hH2Spb3Y