

Homework 9: Dynamical systems & Linear differential equations

Deadline: 26th July, 2020

Exercise 1. (8 Points) Solve the dynamical system $x'(t) = Ax(t)$, where

$$A = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & -1 \\ 0 & 0 & 3 \end{pmatrix} \quad \text{and} \quad x(0) = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}.$$

Exercise 2. (4 Points) Let $F : V \rightarrow W$ be a linear map between two vector spaces V and W . Assume that $F(v) = w$ for a fixed $v \in V$ and $w \in W$. Show that the following two statements are equivalent:

- i) $F(x) = w$.
- ii) $x = v + u$ for some $u \in \ker(F)$.

Exercise 3. (8 Points) Find all solutions to the following differential equation

$$f^{(3)} - 3f'' - 10f' = 10,$$

such that $f(0) = f'(0) = 0$.

(Hint: First, try to find all solutions to the differential equation. For this, find one particular solution (try polynomials) and then consider the homogeneous equation. Then find those solutions out of these such that $f(0) = f'(0) = 0$ holds.)