Homework 7: Eigenvalues & Eigenvectors II

Deadline: 28th June, 2020

Exercise 1. (10 Points) Let $F : V \to V$ be a linear map with eigenvalues $\lambda_1, \dots, \lambda_m \in \mathbb{R}$ and $\dim V = n$. Show the following statements by using Theorem 5.6:

i) If F has n distinct eigenvalues, i.e. $m = n$, then F is diagonalizable.

ii) If $B_1, \dots, B_m$ are bases of $E_{\lambda_1}(F), \dots, E_{\lambda_m}(F)$, then $B_1 \cup \dots \cup B_m$ are linearly independent. (Here we mean by $B_1 \cup \dots \cup B_m$ the collection of all vectors in the bases $B_1, \dots, B_m$.)

iii) The map F is diagonalizable if and only if

$$\sum_{j=1}^{m} \dim E_{\lambda_j}(F) = n \,.$$

Exercise 2. (6 Points) Consider the matrix

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{pmatrix} .$$

Diagonalize the matrix A, i.e. find a matrix S, such that $S^{-1}AS$ is a diagonal matrix.

Exercise 3. (6 Points)

i) Let $U \subset \mathbb{R}^n$ be a subspace and let $P_U : \mathbb{R}^n \to \mathbb{R}^n$ be the orthogonal projection to U. Show that P_U is diagonalizable. What are the eigenvalues of P_U?

ii) Let $\mathrm{rot}_\alpha : \mathbb{R}^2 \to \mathbb{R}^2$ be the rotation by an angle $\alpha \in [0, 2\pi]$. For which α is rot_α diagonalizable?

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