Homework 6: Eigenvalues & Eigenvectors I

Deadline: 21st June, 2020

Exercise 1. (8 Points) Consider the linear map $F : \mathbb{R}^4 \to \mathbb{R}^4$, given by $F(x) = Ax$, where

$$A = \begin{pmatrix} 1 & 1 & 6 & -2 \\ 0 & 1 & -3 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -2 & 2 \end{pmatrix}.$$

i) Determine all eigenvalues of F.

ii) For each eigenvalue λ of F, determine a basis of the eigenspace $E_\lambda(F)$.

Exercise 2. (8 Points) Give examples of linear maps $F_i : \mathbb{R}^3 \to \mathbb{R}^3$ for $i = 1, 2, 3, 4$, such that

i) F_1 has exactly three different eigenvalues $\lambda_1, \lambda_2, \lambda_3 \in \mathbb{R}$.

ii) F_2 has exactly two different eigenvalues $\lambda_1, \lambda_2 \in \mathbb{R}$ and $\dim E_{\lambda_1}(F_2) = \dim E_{\lambda_2}(F_2) = 1$.

iii) F_3 has exactly two different eigenvalues $\lambda_1, \lambda_2 \in \mathbb{R}$ and $\dim E_{\lambda_1}(F_3) = 1$ and $\dim E_{\lambda_2}(F_3) = 2$.

iv) F_4 has exactly one eigenvalue $\lambda_1 \in \mathbb{R}$.

In each case calculate a basis for the eigenspaces.