

Homework 5: Determinants II

Deadline: 7th June, 2020

(The first part of Exercise 1 is Proposition 4.7. in [L])

Exercise 1. (8 Points)

- i) Show that the determinant is linear in each row, i.e. for any $A = (a_{i,j}) \in \mathbb{R}^{n \times n}$ and $1 \leq l \leq n$ show that the map

$$F_{A,l} : \mathbb{R}^n \longrightarrow \mathbb{R} \\ x \longmapsto \det(A(l; x))$$

is linear. Here $A(l; x)$ denotes the matrix A , where the l -th row is replaced by the vector x^T .

- ii) Assume that A is invertible. What is the kernel of $F_{A,n}$?

Exercise 2. (8 Points) Calculate the determinants of the following linear maps.

- i) The determinant of the linear map $F : \mathcal{P}_2 \rightarrow \mathcal{P}_2$ defined for a $p \in \mathcal{P}_2$ by

$$F(p) = p(-1) + p(0)x + p(1)x^2.$$

- ii) The determinant of the linear map $G : \mathcal{P}_n \rightarrow \mathcal{P}_n$ defined by $G(p) = q$, where $q(x) = xp'(x) + p(x)$.

References

[L] Linear Algebra II - Overview notes, See https://www.henrikbachmann.com/la2_2020.html.