Homework 2: Linear maps

Deadline: 17th May, 2020

Recall: $\mathcal{P}_n$ is the vector space of polynomial functions of degree $\leq n$.

Exercise 1. (12 Points) For $n \geq 1$ we define the map $F_n : \mathcal{P}_n \to \mathbb{R}^3$ for a $p \in \mathcal{P}_n$ by

$$F_n(p) = \begin{pmatrix} p(-1) \\ p(0) \\ p(1) \end{pmatrix} .$$

i) Show that F_n is a linear map.

ii) Show that F_2 is invertible, i.e. an isomorphism.

iii) Calculate the inverse of F_2.

iv) Check if F_1 and F_3 are injective and/or surjective.

v) Determine a basis of $\mathrm{im}(F_1)$.

Exercise 2. (8 Points) We define the map $G : \mathbb{R}^3 \to \mathcal{P}_2$ by

$$G \begin{pmatrix} a \\ b \\ c \end{pmatrix} = a + b(1 - x^2) + cx^2 .$$

i) Show that G is a linear map.

ii) Calculate a basis for $\ker(G)$ and $\mathrm{im}(G)$ and determine their dimensions.

Version: May 8, 2020