# Quiz 1: Linear systems – Solutions

Tutorial 1, Tuesday, 6th October, 2026

**Notation.** The row operations are written as in the lecture: a circled number $c$ next to an equation with an arrow to another equation means that $c$ times the first equation is added to the other one (R1); a circled number without an arrow means that the equation is multiplied with this number (R2); a double arrow means that two equations are exchanged (R3).

**Exercise 1.** (3+7 = 10 Points) Consider the linear system

$$\begin{cases} x_1 + x_2 + x_3 = 5 \\ 2x_1 + 3x_2 = 8 \\ x_1 + 2x_2 - x_3 = 3 \end{cases}$$

(i) (3 Points) Which of the following are solutions of this linear system?

(a) $x_1 = 1$, $x_2 = 2$, $x_3 = 2$ (b) $x_1 = 3$, $x_2 = 1$, $x_3 = 1$

(ii) (7 Points) Find all solutions of this linear system.

*Solution.* **(i)** (a) $1 + 2 + 2 = 5$ ✓, $2 \cdot 1 + 3 \cdot 2 = 8$ ✓ and $1 + 2 \cdot 2 - 2 = 3$ ✓: a solution.
(b) $3 + 1 + 1 = 5$ ✓, but $2 \cdot 3 + 3 \cdot 1 = 9 \neq 8$: **not** a solution.

**Answer:** (a) yes, (b) no.

**(ii)** First we eliminate $x_1$ from the second and third equation, then $x_2$ from the third equation:

$$\begin{cases} x_1 + x_2 + x_3 = 5 \\ 2x_1 + 3x_2 = 8 \\ x_1 + 2x_2 - x_3 = 3 \end{cases} \overset{\textcircled{-1}\ \textcircled{-2}}{\Leftrightarrow} \begin{cases} x_1 + x_2 + x_3 = 5 \\ x_2 - 2x_3 = -2 \\ x_2 - 2x_3 = -2 \end{cases} \overset{\textcircled{-1}}{\Leftrightarrow} \begin{cases} x_1 + x_2 + x_3 = 5 \\ x_2 - 2x_3 = -2 \\ 0 = 0 \end{cases}$$

The equation $0 = 0$ can be omitted (the third equation was the second minus the first one), and we get

$$\begin{cases} x_1 + 3x_3 = 7 \\ x_2 - 2x_3 = -2 \end{cases}$$

The variable $x_3$ can be chosen arbitrarily, $x_3 = t$ with $t \in \mathbb{R}$:

**Answer:** infinitely many solutions: $x_1 = 7 - 3t$, $x_2 = -2 + 2t$, $x_3 = t$ for $t \in \mathbb{R}$.

(For $t = 2$ we get the solution (a) of (i).)