Tutorial 11: Linear independence & Bases

(i) Vectors $v_1, \dots, v_l \in \mathbb{R}^n$ are called **linearly independent** if the equation

$$\lambda_1 v_1 + \cdots + \lambda_l v_l = 0 \tag{0.1}$$

with $\lambda_1, \dots, \lambda_l \in \mathbb{R}$ just has the unique solution $\lambda_1 = \cdots = \lambda_l = 0$.

(ii) If there exist another solution of (0.1), i.e. where at least for one $j = 1, \dots, l$ we have $\lambda_j \neq 0$, then the vectors $v_1, \dots, v_l$ are called **linearly dependent**.

Let $V \subset \mathbb{R}^n$ be a subspace. Vectors $v_1, \dots, v_l \in V$ form a **basis of V** if

(i) $V = \operatorname{span}\{v_1, \dots, v_l\}$,

(ii) $v_1, \dots, v_l$ are linearly independent.

In this case, we say that $\{v_1, \dots, v_l\}$ is a basis of V.

Exercise 1. Consider the following linear map

$$\begin{aligned} F : \mathbb{R}^5 &\longrightarrow \mathbb{R}^3 \\ x &\longmapsto \begin{pmatrix} -2 & 2 & 2 & 0 & 6 \\ -2 & 2 & 1 & -3 & 5 \\ -3 & 3 & 2 & -3 & 8 \end{pmatrix} x \,. \end{aligned}$$

(i) Find a basis for $\ker(F)$.

(ii) Find a basis for $\operatorname{im}(F)$.

Plan for the coming weeks:

(i) Friday 20th December during the lecture: Christmath Challenge 2023 (45minutes) & Lecture 11 (45 minutes). Make sure to be on time and bring your phone or laptop to take the challenge (we will again use menti.com). Content of the challenge: Lecture 1 - 10.

(ii) Last meeting this year: Next Tuesday 24th December we have tutorial and will talk about Homework 6.

(iii) First meeting next year: Friday 10th January for Lecture 12.