

Homework 7: Orthogonality & Least squares

Deadline: 28th January, 2025

Exercise 1. (6 Points) Let $U = \text{span}\{u_1, u_2, u_3, u_4\} \in \mathbb{R}^4$, where

$$u_1 = \begin{pmatrix} 1 \\ -2 \\ 3 \\ 0 \end{pmatrix}, u_2 = \begin{pmatrix} 2 \\ -2 \\ 4 \\ 2 \end{pmatrix}, u_3 = \begin{pmatrix} 2 \\ 0 \\ 2 \\ 4 \end{pmatrix}, u_4 = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 4 \end{pmatrix}.$$

- (i) Determine a basis $B = (b_1, \dots, b_m)$ of U .
- (ii) Calculate the coordinate vectors $[u_j]_B \in \mathbb{R}^m$ for $j = 1, 2, 3, 4$.

Exercise 2. (6 Points) We define the following vectors

$$b_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix}, b_2 = \begin{pmatrix} 1 \\ 2 \\ 0 \\ 1 \end{pmatrix}, b_3 = \begin{pmatrix} 1 \\ 2 \\ 1 \\ 3 \end{pmatrix}.$$

These form a basis $B = (b_1, b_2, b_3)$ of the subspace $U = \text{span}\{b_1, b_2, b_3\} \subset \mathbb{R}^4$ (You do not need to show this). Use the Gram-Schmidt algorithm to construct an orthonormal basis $F = (f_1, f_2, f_3)$ of U from B .

Exercise 3. (8 Points) Assume we have the following data points

i	1	2	3	4
x_i	0	1	2	3
y_i	-2	1	2	4

- (i) Find the line of best fit for the above data, that is, find $a, b \in \mathbb{R}$ such that the function $l(x) = ax + b$ minimizes the sum of squares $\sum_{i=1}^4 (l(x_i) - y_i)^2$.
- (ii) Interpolate the data by a quadratic polynomial. For this find $c, d, e \in \mathbb{R}$ such that the function $p(x) = cx^2 + dx + e$ minimizes $\sum_{i=1}^4 (p(x_i) - y_i)^2$.
- (iii) Draw the data points and the graphs of l and p into one diagram.

For both (i) and (ii) solve the exercise by finding the solutions to the normal equation.

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Happy New Year! This is the last HW for LA1, and therefore this is the last Japanese corner.

Today's words are some of the ones used in this homework!

さいしょう に じょうほう
最小二乗法

せい き ちよっこう きてい
正規直交基底

These words are: saishou nijouhou (**least-square method**) and seikichokkou kitei (**orthonormal basis**). Today's words (and the last ones for this semester) are:

さい
最

- This kanji means "(**the**) **most**". It, combined with 小 will make ^{さいしょう}最小, meaning "smallest". A common use is in ^{さいきん}最近 (recently).

じょう
乗

- This kanji means "**to ride**". A common example will be found when one travels and must change lines (e.g. JR or subway), namely ^の乗り換え (interchange).

せい
正

- This kanji means "**right**". Here, ^{せい き}正規 means "normal". This kanji is used in ^{せいぎ}正義 (justice), ^{せいかい}正解 (correct answer), and ^{しゅうせい}修正 (edit).

き
規

- This kanji (combined with 正) means "**conform (to)**". This kanji is a very uncommon kanji.

ちよっ
直

- This kanji means "**straight**". In this context, ^{ちよっこう}直交 means "orthogonal". This kanji is used in ^{ちよくせつ}直接 (upfront) and ^{しょうじき}正直 (honestly).

こう
交

- This kanji means "**swap**". This kanji is used in ^{こうさてん}交差点 (crossroads) and ^{こうたい}交代 (to exchange).

And that's it for today. I wish you luck in the finals. Thank you for reading this Japanese corner.