Homework 5: Subspaces

Deadline: 24th December, 2024

Exercise 1. (5+3 = 8 Points)

(i) Which of the following subsets are subspaces? Justify your answers.

$$\begin{aligned}
U_1 &= \left\{x \in \mathbb{R}^3 \mid x_1^2 + x_2^2 = x_3^3\right\}, \\
U_2 &= \{x \in \mathbb{R}^n \mid Ax = 3x\}, \quad \text{where } A \in \mathbb{R}^{n\times n} \text{ is a fixed matrix}, \\
U_3 &= \left\{x \in \mathbb{R}^4 \mid x_1 + x_2 = x_3 \text{ and } x_2 + x_3 = x_4\right\}, \\
U_4 &= \{x \in \mathbb{R}^n \mid x \bullet v = 0\}, \quad \text{for a fixed } v \in \mathbb{R}^n, \\
U_5 &= \left\{x \in \mathbb{R}^2 \mid x_1 \leq x_2\right\}.
\end{aligned}$$

(ii) For each subset U in (i) that is a subspace, find numbers $a, b \geq 1$ and a linear map $F : \mathbb{R}^a \to \mathbb{R}^b$ such that $\ker(F) = U$.

Exercise 2. (2+3+3 = 8 Points) Consider the following subspace

$$W = \ker(P_u) = \left\{x \in \mathbb{R}^3 \mid P_u(x) = 0\right\}, \quad \text{where } u = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}.$$

(i) Determine vectors $v_1, \ldots, v_m \in \mathbb{R}^3$ with $W = \operatorname{span}\{v_1, \ldots, v_m\}$.

(ii) Find a linear map $H : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ with $\operatorname{im}(H) = W$.

(iii) Calculate $\ker(H)$ and $\ker(P_u \circ H)$.

Exercise 3. (3+3 = 6 Points)

(i) Let $U, V \subset \mathbb{R}^m$ be subspaces. Decide whether the union $U \cup V$ is also a subspace or not.

(ii) Let $U, V \subset \mathbb{R}^m$ be subspaces. Decide whether the intersection $U \cap V$ is also a subspace or not.

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くま先生の 簡単数学用語 解説コーナー

The days are getting short, and the nights are getting cold (in Nagoya). Winter has come.

I hope you guys are ready for Professor Bachmann's Linear Algebra Chrismath Challenge!

Today's words are mostly on the subject of the HW: Inverses and Subspaces.

核 像 可逆 部分空間

These words are: kaku (**kernel**), zou (**image**), kagyaku (**invertible**), and bubun kuukan (**subspace**).

Here's a question: how do we say "invertible matrix" in Japanese?

While I won't give the answer here, interestingly, there are a few Japanese synonyms for "invertible matrix", namely 正則行列 (lit. regular matrix) and 非特異行列 (lit. non-singular matrix).

Note that this time, 像 is omitted due to its inclusion in HW3's part.

- **核** (かく) - Read as "かく". This kanji usually means "**nuclear**" or "**nucleus**". Uses include 原子核 (Atomic Nucleus) and 核兵器 (Nuclear Weapons).
- **可** (か) - This kanji means "**able**" or "**consent**". It refers to how an invertible matrix is "able" to be inverted. Common uses of this kanji include 可能性 (Possibility), 許可 (permission), and 不可 (unable, or "to fail" a course)
- **逆** (ぎゃく) - This kanji means "**reverse**". It refers to how an inverse matrix is a reverse of the original matrix. One use of this kanji in everyday life is in the expression 逆ですよ (meaning, **no, it's the opposite!**).
- **部** (ぶ) - This kanji means "**part**". This word is used in 学部 (Undergraduate). Other uses of this kanji include 部下 (subordinate), 全部 (all), and 部屋 (room).
- **分** (ぶん) - This kanji means "**separate**". One word that you might have encountered that uses this kanji is 分かる (to understand). Other common uses include 気分 (feeling) and 多分 (probably).
- **空** (くう) - This kanji means "**space**" or "**sky**". It refers to how subspaces are... spaces! This kanji is very common in everyday life, with its common uses including 空 (sky), 空港 (airport), 空白 (blank space) and 空手.
- **間** (かん) - This kanji means "**interval**". It refers to how subspaces are "intervals" of space. This kanji is also common in everyday life, used in (among others) 時間 (time), 間 (between), 瞬間 (moment), and 人間 (human).

And that's it for today. Take care, Good Luck, and Have fun (GLHF)!

P.S. In case you didn't guess it yet, "Invertible Matrix" is 可逆行列 (kagyaku gyouretsu).