Linear Algebra I & Math. Tutorial 1b - Fall 2023
Course information

Homepage for this course: `https://www.henrikbachmann.com/la1_2023.html`. Please check this page regularly for updates on the schedule and/or to get information on the course and the Homework.

Preliminary schedule

	Tutorial (1b)	Lecture	Preliminary lecture topic	Section
1	3rd Oct (Tu)	6th Oct (Fr)	Introduction & Linear systems	1
2	10th Oct (Tu)	13th Oct (Fr)	Matrices and vectors	2
3	17th Oct (Tu)	20th Oct (Fr)	Sets and functions	3
4	24th Oct (Tu)	27th Oct (Fr)	Linear maps	4
5	31th Oct (Tu)	- culture day -	Linear maps	4
6	7th Nov (Tu)	10th Nov (Fr)	Linear maps in geometry	5
7	14th Nov (Tu)	**17th Nov (Fr)**	**Midterm exam (in the lecture)**	1-5
7	-	18th Nov (Sa)	Matrix multiplication	6
8	21st Nov (Tu)	24th Nov (Fr)	The inverse of a linear map	7
9	28th Nov (Tu)	1st Dec (Fr)	Subspaces, Kernel & Image	8
10	5th Dec (Tu)	8th Dec (Fr)	Linear indepedence & Bases I	9,10
11	12th Dec (Tu)	15th Dec (Fr)	Bases II & Dimension	10
12	19th Dec (Tu)	22nd Dec (Fr)	Coordinates, 🎄 Christmath challenge 🎄	11
13	-	10th Jan (We)	Coordinates & Orthogonal bases	12
14	16th Jan (Tu)	19th Jan (Fr)	Orthogonal projection	13
15	23rd Jan (Tu)	26th Jan (Fr)	Review	7-13
16	30th Jan (Tu)	**2nd Feb (Fr)**	**Final exam (in the lecture)**	1-13

Times and venues

- All **lectures** take place in the room **C15** in the Liberal Arts and Sciences building, between 13:00 and 14:30. All **tutorials** take place in the room **A407** in Science building A, between 13:00 and 13:45. (From 13:45 there will be the Calculus tutorial in the same room)

- Before the midterm and final exams, I offer question sessions in Zoom. We will decide together on the day and time for this.

Contact

Henrik Bachmann, Office: Graduate School of Mathematics Building, Room: 457
(http://en.nagoya-u.ac.jp/map/index.html, Building D3-4)

E-mail: **henrik.bachmann@math.nagoya-u.ac.jp**
Homepage: `http://www.henrikbachmann.com`

Please feel free to contact me at any time via email or come directly to my office. There are no "stupid questions," and asking a lot of questions will not influence your grade in any way.

Textbook & Notes

Every student is encouraged to take his/her personal notes during the lecture and tutorial. We will provide lecture notes and update it after each lecture. There also exists a reference book: Otto Bretscher: *Linear Algebra with Applications*, 4th edition, Pearson 2009. (available at the Central and Science libraries). You will not need to get this book, but it contains a lot of practice Exercises.

Examination

The examination consists of a **midterm exam** and a **final exam**, together with **homework**.

- The midterm exam is held on the 17th of November, 2023, and the final exam is on the 2nd of February, 2024. Both will take place in the lecture room C15 during the usual lecture time.

- **Homework :** There will be a number of homework assignments during the course. Collaboration is encouraged, but solutions must be written and handed in individually. You can write down your solutions by hand (paper, tablet) <u>or</u> by computer (Latex only. No word!). Write your name, the homework number and the course name on the first page of your solution. Create **one pdf-file** (for example, by using a scanner app on your phone) and submit it before the deadline ends in **TACT** at the corresponding Assignment. Use exactly the following format as a filename:

  ”**Familyname_Givenname_LA1_HWX.pdf**”,

  where X stands for the number of the Homework. We will remove points if you do not follow these simple rules.

- **Repeat exam:** There will be a repeat exam during the winter vacation for those who failed the ordinary examination. This will take place somewhere in March 2023.

Grading

A total score T (0–100 %) is calculated as the weighted average of the percentages obtained from the homework H (0–100 %), midterm exam M (0–100 %) and final exam F (0–100 %) as follows

$$T = \alpha_H H + \alpha_M M + \alpha_F F.$$

Here the weights $\alpha_H, \alpha_M, \alpha_F$ can be determined from the following information:

A student who ..

1. .. got 70% in the Homework, 70% in the midterm, and 75% in the final, gets a total score of 72%.

2. .. got 80% in the Homework, 80% in the midterm, and 85% in the final, gets a total score of 82%.

3. .. got 100% in the Homework, 80% in the midterm, and 95% in the final, gets a total score of 92%.

What are the weights $\alpha_H, \alpha_M, \alpha_F$?

The total score will be used to determine a grade A+, A, B, C, C-, or F. (The exact grading scheme will be determined after the final exam). This grade will be the final grade for **both** ”Linear Algebra I” and ”Math. Tutorial 1b”. If you plan to just take one of these courses, please contact me (This is possible.).

Tutorial Exercise

A total score T (0–100 %) is calculated as the weighted average of the percentages obtained from the homework H (0–100 %), midterm exam M (0–100 %) and final exam F (0–100 %) as follows

$$T = \alpha_H H + \alpha_M M + \alpha_F F.$$

Here the weights $\alpha_H, \alpha_M, \alpha_F$ can be determined from the following information:

A student who ..

1. .. got 70% in the Homework, 70% in the midterm, and 75% in the final, gets a total score of 72%.

2. .. got 80% in the Homework, 80% in the midterm, and 85% in the final, gets a total score of 82%.

3. .. got 100% in the Homework, 80% in the midterm, and 95% in the final, gets a total score of 92%.

What are the weights $\alpha_H, \alpha_M, \alpha_F$?

For each student we write down a __linear equation__

1. $\qquad 70\alpha_H + 70\alpha_M + 75\alpha_F = 72$

2. $\qquad 80\alpha_H + 80\alpha_M + 85\alpha_F = 82$

3. $\qquad 100\alpha_H + 80\alpha_M + 95\alpha_F = 92$

Such a collection of linear equations is called a __linear system__.

Goal: Find $\alpha_H, \alpha_M, \alpha_F$ such that they satisfy all lin. eq. (This is called a solution of the linear system).

Strategy: Add/Subtract multiples of one equation to another to get an easier __equivalent__ linear system. Meaning they have the same solutions

(-1) (-1)

$$\begin{cases} 70\,\alpha_H + 70\alpha_M + 75\alpha_F = 72 \\ 80\,\alpha_H + 80\alpha_M + 85\alpha_F = 82 \\ 100\,\alpha_H + 80\alpha_M + 95\alpha_F = 92 \end{cases}$$

$\Longleftrightarrow$ $\frac{1}{10}$ $\frac{1}{10}$

$$\begin{cases} 70\,\alpha_H + 70\alpha_M + 75\alpha_F = 72 \\ 10\,\alpha_H + 10\,\alpha_M + 10\alpha_F = 10 \\ 20\alpha_H + 0 + 10\,\alpha_F = 10 \end{cases}$$

equivalent
"if and only if"

By this we mean that we subtract the first eq. from the second and the second from the third

$\Longrightarrow$ (-2) (70)

$$\begin{cases} 70\,\alpha_H + 70\alpha_H + 75\alpha_F = 72 \\ \alpha_H + \alpha_M + \alpha_F = 1 \\ 2\alpha \qquad + \alpha_F = 1 \end{cases}$$

$(\frac{1}{5})$

$\Longleftrightarrow$ $(-\frac{1}{2})$ $(\frac{1}{2})$

$$\begin{cases} 5\alpha_F = 2 \\ \alpha_H + \alpha_M + \alpha_F = 1 \\ -2\alpha_M - \alpha_F = -1 \end{cases}$$

$(-\frac{1}{2})$

$\Longleftrightarrow$

$$\begin{cases} \alpha_F = \dfrac{2}{5} \\ \alpha_H + \dfrac{1}{2}\alpha_F = \dfrac{1}{2} \\ \alpha_M + \dfrac{1}{2}\alpha_F = \dfrac{1}{2} \end{cases}$$

$$\frac{1}{2} - \frac{1}{2}\cdot\frac{2}{5} = \frac{5}{10} - \frac{2}{10} = \frac{3}{10}$$

$\Longleftrightarrow$

$$\begin{cases} \alpha_F = \dfrac{2}{5} \\ \alpha_H = \dfrac{3}{10} \\ \alpha_M = \dfrac{3}{10} \end{cases}$$

$\Longrightarrow$ Home work & Midterm count 30% and the final 40%