

## Homework 7: Orthogonality & Least squares

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Deadline: 28th January, 2024

**Exercise 1.** (6 Points) Let  $U = \text{span}\{u_1, u_2, u_3, u_4\} \in \mathbb{R}^4$ , where

$$u_1 = \begin{pmatrix} 3 \\ -3 \\ 5 \\ 7 \end{pmatrix}, \quad u_2 = \begin{pmatrix} 0 \\ 1 \\ 4 \\ 6 \end{pmatrix}, \quad u_3 = \begin{pmatrix} 3 \\ -2 \\ 9 \\ 13 \end{pmatrix}, \quad u_4 = \begin{pmatrix} 3 \\ -4 \\ 1 \\ 1 \end{pmatrix}.$$

- (i) Determine a basis  $B = (b_1, \dots, b_m)$  of  $U$ .
- (ii) Calculate the coordinate vectors  $[u_j]_B \in \mathbb{R}^m$  for  $j = 1, 2, 3, 4$ .

**Exercise 2.** (6 Points) We define the following vectors

$$b_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix}, \quad b_2 = \begin{pmatrix} 1 \\ 2 \\ 0 \\ 1 \end{pmatrix}, \quad b_3 = \begin{pmatrix} 1 \\ 2 \\ 1 \\ 3 \end{pmatrix}.$$

These form a basis  $B = (b_1, b_2, b_3)$  of the subspace  $U = \text{span}\{b_1, b_2, b_3\} \subset \mathbb{R}^4$  (You do not need to show this). Use the Gram-Schmidt algorithm to construct an orthonormal basis  $F = (f_1, f_2, f_3)$  of  $U$  from  $B$ .

**Exercise 3.** (8 Points) Assume we have the following data points

$i$	1	2	3	4
$x_i$	0	1	2	3
$y_i$	2	1	3	4

- (i) Find the line of best fit for the above data, that is, find  $a, b \in \mathbb{R}$  such that the function  $l(x) = ax + b$  minimizes the sum of squares  $\sum_{i=1}^4 (l(x_i) - y_i)^2$ .
- (ii) Interpolate the data by a quadratic polynomial. For this find  $c, d, e \in \mathbb{R}$  such that the function  $p(x) = cx^2 + dx + e$  minimizes  $\sum_{i=1}^4 (p(x_i) - y_i)^2$ .
- (iii) Draw the data points and the graphs of  $l$  and  $p$  into one diagram.

For both (i) and (ii) solve the exercise by finding the solutions to the normal equation (Tutorial 14).

くま先生の  
**簡単数学用語**  
**解説コーナー**



Happy New Year! This is the last HW for LA1, and therefore this is the last Japanese corner.

Today's words are some of the ones used in this homework!

さいしょう に じょうほう  
**最小二乗法**

せい き ちよっこう きてい  
**正規直交基底**

These words are: saishou nijouhou (**least-square method**) and seikichokkou kitei (**orthonormal basis**). Today's words (and the last ones for this semester) are:

さい  
**最**

- This kanji means "(**the**) **most**". It, combined with 小 will make <sup>さいしょう</sup>最小, meaning "smallest". A common use is in <sup>さいきん</sup>最近 (recently).

じょう  
**乗**

- This kanji means "**to ride**". A common example will be found when one travels and must change lines (e.g. JR or subway), namely <sup>の</sup>乗り換え (interchange).

せい  
**正**

- This kanji means "**right**". Here, <sup>せい き</sup>正規 means "normal". This kanji is used in <sup>せいぎ</sup>正義 (justice), <sup>せいかい</sup>正解 (correct answer), and <sup>しゅうせい</sup>修正 (edit).

き  
**規**

- This kanji (combined with 正) means "**conform (to)**". This kanji is a very uncommon kanji.

ちよっ  
**直**

- This kanji means "**straight**". In this context, <sup>ちよっこう</sup>直交 means "orthogonal". This kanji is used in <sup>ちよくせつ</sup>直接 (upfront) and <sup>しょうじき</sup>正直 (honestly).

こう  
**交**

- This kanji means "**swap**". This kanji is used in <sup>こうさてん</sup>交差点 (crossroads) and <sup>こうたい</sup>交代 (to exchange).

And that's it for today. I wish you luck in the finals. Thank you for reading this Japanese corner.