# Tutorial 4: Functions

Let $X$ and $Y$ be two sets. For a function $f : X \to Y$ the **image of** $f$ is defined by

$$\operatorname{im}(f) = f(X) = \{y \in Y \mid \exists x \in X : y = f(x)\} \subset Y\,.$$

**Exercise 1.** Calculate the image of the following functions:

$$\begin{aligned} f_1 : \mathbb{R} &\longrightarrow \mathbb{R} \\ x &\longmapsto e^x - 3\,, \end{aligned} \qquad \begin{aligned} f_2 : \mathbb{R} &\longrightarrow \mathbb{R} \\ x &\longmapsto x^2 + 1\,, \end{aligned}$$

$$\begin{aligned} f_3 : \mathbb{R}^2 &\longrightarrow \mathbb{R}^2 \\ \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} &\longmapsto \begin{pmatrix} x_1 + 2x_2 \\ 2x_1 + 4x_2 \end{pmatrix}\,, \end{aligned} \qquad \begin{aligned} f_4 : \mathbb{R}^2 &\longrightarrow \mathbb{R}^2 \\ \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} &\longmapsto \begin{pmatrix} x_1 + 2x_2 \\ 3x_1 + 4x_2 \end{pmatrix}\,. \end{aligned}$$

A function $f : X \to Y$ is called

i) **injective** if $f(x_1) \neq f(x_2)$ whenever $x_1 \neq x_2$. $(x_1, x_2 \in X)$

ii) **surjective** if $\operatorname{im}(f) = Y$.

iii) **bijective** if it is injective and surjective.

**Exercise 2.** For $X = \mathbb{R}$ try to find an example of a function $f : X \to X$, which is

i) not injective and not surjective.

ii) injective but not surjective.

iii) not injective but surjective.

iv) bijective.

Do the same for $X = \{1, 2, 3\}$. Is it possible to find examples for all cases?

**Exercise 3.** We define for a matrix $A \in \mathbb{R}^{m \times n}$ the function

$$\begin{aligned} f : \mathbb{R}^n &\longrightarrow \mathbb{R}^m \\ x &\longmapsto Ax\,. \end{aligned}$$

Find again examples of $m$, $n$ and $A$ for the cases i),ii),iii) and iv) as in Exercise 4.