Tutorial 11: Linear independence & Bases

(i) Vectors $v_1, \dots, v_l \in \mathbb{R}^n$ are called **linearly independent** if the equation

$$\lambda_1 v_1 + \cdots + \lambda_l v_l = 0 \tag{0.1}$$

with $\lambda_1, \dots, \lambda_l \in \mathbb{R}$ just has the unique solution $\lambda_1 = \cdots = \lambda_l = 0$.

(ii) If there exist another solution of (0.1), i.e. where at least for one $j = 1, \dots, l$ we have $\lambda_j \neq 0$, then the vectors $v_1, \dots, v_l$ are called **linearly dependent**.

Let $V \subset \mathbb{R}^n$ be a subspace. Vectors $v_1, \dots, v_l \in V$ form a **basis of V** if

i) $V = \operatorname{span}\{v_1, \dots, v_l\}$,

ii) $v_1, \dots, v_l$ are linearly independent.

In this case we also say that $\{v_1, \dots, v_l\}$ is a basis of V.

Exercise 1. Consider the following linear maps

$$\begin{aligned} F : \mathbb{R}^5 &\longrightarrow \mathbb{R}^3 \\ x &\longmapsto \begin{pmatrix} -2 & 2 & 2 & 0 & 6 \\ -2 & 2 & 1 & -3 & 5 \\ -3 & 3 & 2 & -3 & 8 \end{pmatrix} x\,. \end{aligned}$$

i) Find a basis for $\ker(F)$.

ii) Find a basis for $\operatorname{im}(F)$.

Plan for the coming weeks:

(i) Friday 23rd December during the lecture: Christmath Challenge 2022 (45minutes) & Lecture 11 (45 minutes). Make sure to be on time and to bring your phone or laptop to take the challenge (we will use again menti.com). Content of the challenge: Lecture 1 - 10 (and more...)

(ii) Tuesday 27th December: No tutorials (Also no Calculus tutorial)

(iii) Tuesday 10th January: **Usual tutorial in A408** (Even though this is a make-up day for Friday).

(iv) Friday 13th January: Lecture 12 online in Zoom. On this day there is the "National Center Test for Japanese University Admissions" and we can not use the lecture hall. This lecture will be recorded.