

Tutorial 1

Linear Algebra I & Math. Tutorial 1b - Fall 2022

Course information

Homepage for this course: https://www.henrikbachmann.com/la1_2022.html. Please check this page regularly for updates on the schedule and/or to get information on the course and the Homework.

Preliminary schedule

	Tutorial (1b)	Lecture	Preliminary lecture topic	Section
1	4th Oct (Tu)	7th Oct (Fr)	Introduction & Linear systems	1
2	11th Oct (Tu)	14th Oct (Fr)	Matrices and vectors	2
3	18th Oct (Tu)	21th Oct (Fr)	Sets and functions	3
4	25th Oct (Tu)	28th Oct (Fr)	Linear maps	4
5	1st Nov (Tu)	4th Nov (Fr)	Linear maps in geometry	5
6	8th Nov (Tu)	11th Nov (Fr)	Matrix multiplication	6
7	15th Nov (Tu)	18th Nov (Fr)	Midterm exam (in the lecture)	1-6
8	22nd Nov (Tu)	25th Nov (Fr)	The inverse of a linear map	7
9	-	2nd Dec (Fr)	Subspaces, Kernel & Image	8
10	6th Dec (Tu)	9th Dec (Fr)	Linear independence & Bases I	9,10
11	13th Dec (Tu)	16th Dec (Fr)	Bases II & Dimension	10
12	20th Dec (Tu)	23rd Dec (Fr)	Christmath challenge & Coordinates	11
13	10th Jan (Tu)	13th Jan (Fr)	Coordinates & Orthogonal bases	12
14	17th Jan (Tu)	20th Jan (Fr)	Orthogonal projection	13
15	24th Jan (Tu)	27th Jan (Fr)	Review	7-13
16	31st Jan (Tu)	3rd Feb (Fr)	Final exam (in the lecture)	1-13

Times and venues

- All **lectures** take place in the room **C23** in the Liberal Arts and Sciences building, between 13:00 and 14:30. All **tutorials** take place in the room **A402** in Science building A, between 13:00 and 13:45. (From 13:45 there will be the Tutorial 1a (Calculus) in the Room A407).
- Before the midterm and final exams, I offer question sessions in Zoom. We will decide together on the day and time for this.

Contact

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Please feel free to contact me at any time via email or come directly to my office. There are no "stupid questions," and asking a lot of questions will not influence your grade in any way.

Textbook & Notes

Otto Bretscher: *Linear Algebra with Applications*, 4th edition, Pearson 2009.

(available at the Central and Science libraries).

You will not need to get this book, but it contains a lot of practice Exercises.

On the homepage you can find overview notes for this courses and handwritten lectures notes (uploaded after the lecture). But every student is encouraged to take his/her personal notes.

Examination

The examination consists of a **midterm exam** and a **final exam**, together with **homework**.

- The midterm exam is held on the 18th of November, 2022, and the final exam is on the 3rd of February, 2023. Both will take place in the lecture room C23 during the usual lecture time.
- **Homework** : There will be a number of homework assignments during the course. Collaboration is encouraged, but solutions must be written and handed in individually. You can write down your solutions by hand (paper, tablet) **or** by computer (Latex only. No word!). Write your name, the homework number and the course name on the first page of your solution. Create **one pdf-file** (for example, by using a scanner app on your phone) and submit it before the deadline ends in **NUCT** at the corresponding Assignment. Use exactly the following format as a filename: **"Familyname_Givenname_LA1_HWX.pdf"**, where X stands for the number of the Homework. We will remove points if you do not follow these simple rules.
- **Repeat exam**: There will be a repeat exam during the winter vacation for those who failed the ordinary examination. This will take place somewhere in March 2023.

Grading

A total score T (0–100 %) is calculated as the weighted average of the percentages obtained from the homework H (0–100 %), midterm exam M (0–100 %) and final exam F (0–100 %) as follows

$$T = \alpha_H H + \alpha_M M + \alpha_F F.$$

Here the weights $\alpha_H, \alpha_M, \alpha_F$ can be determined from the following information:

A student who ..

1. .. got 70% in the Homework, 70% in the midterm, and 75% in the final, gets a total score of 72%.
2. .. got 80% in the Homework, 80% in the midterm, and 85% in the final, gets a total score of 82%.
3. .. got 100% in the Homework, 80% in the midterm, and 95% in the final, gets a total score of 92%.

What are the weights $\alpha_H, \alpha_M, \alpha_F$?

The total score will be used to determine a grade A+, A, B, C, C-, or F. (The exact grading scheme will be determined after the final exam). This grade will be the final grade for **both** "Linear Algebra I" and "Math. Tutorial 1b". If you plan to just take one of these courses, please contact me (This is possible).

Tutorial Exercise

A total score T (0–100 %) is calculated as the weighted average of the percentages obtained from the homework H (0–100 %), midterm exam M (0–100 %) and final exam F (0–100 %) as follows

$$T = \alpha_H H + \alpha_M M + \alpha_F F.$$

Here the weights $\alpha_H, \alpha_M, \alpha_F$ can be determined from the following information:

A student who ..

1. .. got 70% in the Homework, 70% in the midterm, and 75% in the final, gets a total score of 72%.
2. .. got 80% in the Homework, 80% in the midterm, and 85% in the final, gets a total score of 82%.
3. .. got 100% in the Homework, 80% in the midterm, and 95% in the final, gets a total score of 92%.

What are the weights $\alpha_H, \alpha_M, \alpha_F$?

For each student we write down a linear equation

1. $70\alpha_H + 70\alpha_M + 75\alpha_F = 72$
2. $80\alpha_H + 80\alpha_M + 85\alpha_F = 82$
3. $100\alpha_H + 80\alpha_M + 95\alpha_F = 92$

Such a collection of linear equations is called a linear system.

Goal: Find $\alpha_H, \alpha_M, \alpha_F$ such that they satisfy all lin. eq. (This is called a solution of the linear system).

Strategy: Add/Subtract multiplier of one equation to another to get an easier equivalent linear system.
 Meaning they have the same solutions

We write this to indicate
that all lin. eq. are part of one lin. system

$$\left\{ \begin{array}{l} 70\alpha_H + 70\alpha_M + 75\alpha_F = 72 \\ 80\alpha_H + 80\alpha_M + 85\alpha_F = 82 \\ 100\alpha_H + 80\alpha_M + 95\alpha_F = 92 \end{array} \right. \Leftrightarrow \left\{ \begin{array}{l} 70\alpha_H + 70\alpha_M + 75\alpha_F = 72 \\ 10\alpha_H + 10\alpha_M + 10\alpha_F = 10 \\ 20\alpha_H + 0 + 10\alpha_F = 10 \end{array} \right.$$

equivalent
"if and only if"

By this we mean that we subtract the first eq. from the second and the second from the third

$$\Leftrightarrow \left\{ \begin{array}{l} 70\alpha_H + 70\alpha_M + 75\alpha_F = 72 \\ \alpha_H + \alpha_M + \alpha_F = 1 \\ 2\alpha_H + \alpha_F = 1 \end{array} \right.$$

$$\Leftrightarrow \left\{ \begin{array}{l} 5\alpha_F = 2 \\ \alpha_H + \alpha_M + \alpha_F = 1 \\ -2\alpha_M - \alpha_F = -1 \end{array} \right.$$

$$\Leftrightarrow \left\{ \begin{array}{l} \alpha_F = \frac{2}{5} \\ \alpha_H + \frac{1}{2}\alpha_F = \frac{1}{2} \\ \alpha_M + \frac{1}{2}\alpha_F = \frac{1}{2} \end{array} \right.$$

$$\frac{1}{2} - \frac{1}{2} \cdot \frac{2}{5} = \frac{5}{10} - \frac{2}{10} = \frac{3}{10}$$

$$\Leftrightarrow \left\{ \begin{array}{l} \alpha_H = \frac{3}{10} \\ \alpha_M = \frac{3}{10} \\ \alpha_F = \frac{2}{5} \end{array} \right.$$

\Rightarrow Homework & Midterm count 30% and the final 40%