

Homework 7: Orthogonality

Deadline: 29th January, 2023

Exercise 1. (3+3=6 Points) Let $U = \text{span}\{u_1, u_2, u_3, u_4\} \in \mathbb{R}^4$, where

$$u_1 = \begin{pmatrix} 3 \\ -4 \\ 1 \\ -3 \end{pmatrix}, \quad u_2 = \begin{pmatrix} 3 \\ -3 \\ 5 \\ 2 \end{pmatrix}, \quad u_3 = \begin{pmatrix} 0 \\ 1 \\ 4 \\ 5 \end{pmatrix}, \quad u_4 = \begin{pmatrix} 3 \\ -2 \\ 9 \\ 7 \end{pmatrix}.$$

- i) Determine a basis $B = (b_1, \dots, b_m)$ of U .
- ii) Calculate the coordinate vectors $[u_j]_B \in \mathbb{R}^m$ for $j = 1, 2, 3, 4$.

Exercise 2. (1+1+2+2=6 Points) Let $U \subset \mathbb{R}^n$ be a subspace with orthonormal basis (f_1, \dots, f_r) . We define the orthogonal projection onto U by

$$P_U : \mathbb{R}^n \longrightarrow \mathbb{R}^n \\ x \longmapsto \sum_{i=1}^r (x \bullet f_i) f_i.$$

Show the following properties of P_U :

- (i) If $U = \text{span}\{u\}$ with $u \in \mathbb{R}^n$ and $u \neq 0$ then P_U is the projection P_u we defined in Section 5.
- (ii) P_U is a linear map.
- (iii) $P_U \circ P_U = P_U$.
- (iv) $\text{im } P_U = U$ and $\ker(P_U) = U^\perp$, where U^\perp is the orthogonal complement of U defined by

$$U^\perp = \{x \in \mathbb{R}^n \mid x \bullet u = 0 \text{ for all } u \in U\}.$$

Exercise 3. (6 Points) We define the following vectors

$$b_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix}, \quad b_2 = \begin{pmatrix} 1 \\ 2 \\ 0 \\ 1 \end{pmatrix}, \quad b_3 = \begin{pmatrix} 1 \\ 2 \\ 1 \\ 3 \end{pmatrix}.$$

These form a basis $B = (b_1, b_2, b_3)$ of the subspace $U = \text{span}\{b_1, b_2, b_3\} \subset \mathbb{R}^4$ (You do not need to show this). Use the Gram-Schmidt algorithm to construct an orthonormal basis $F = (f_1, f_2, f_3)$ of U from B .

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Happy New Year! This is the last HW for LA1, and therefore this is the last Japanese corner.

Today's words are some of the ones used in this homework!

さいしょう に じょうほう
最小二乗法

せい き ちよっこう きてい
正規直交基底

These words are: saishou nijouhou (**least-square method**) and seikichokkou kitei (**orthonormal basis**). Today's words (and the last ones for this semester) are:

さい
最

- This kanji means "(the) **most**". It, combined with 小 will make 最小, meaning "smallest". A common use is in 最近 (recently).

じょう
乗

- This kanji means "**to ride**". A common example will be found when one travels and must change lines (e.g. JR or subway), namely 乗り換え (interchange).

せい
正

- This kanji means "**right**". Here, 正規 means "normal". This kanji is used in 正義 (justice), 正解 (correct answer), and 修正 (edit).

き
規

- This kanji (combined with 正) means "**conform (to)**". This kanji is a very uncommon kanji.

ちよっ
直

- This kanji means "**straight**". In this context, 直交 means "orthogonal". This kanji is used in 直接 (upfront) and 正直 (honestly).

こう
交

- This kanji means "**swap**". This kanji is used in 交差点 (crossroads) and 交代 (to exchange).

And that's it for today. I wish you luck in the finals. Thank you for reading this Japanese corner.