

## Homework 4: Linear maps II & Inverses

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Deadline: 4th December, 2022

**Exercise 1.** (3+5 = 8 Points) Let  $u = \begin{pmatrix} u_1 \\ \vdots \\ u_n \end{pmatrix} \in \mathbb{R}^n$  be with  $u \neq 0$ .

- i) Show that the reflection  $\rho_u : \mathbb{R}^n \rightarrow \mathbb{R}^n$  is a linear map.
- ii) Show that the matrix of the projection  $P_u : \mathbb{R}^n \rightarrow \mathbb{R}^n$  is given by

$$[P_u] = \frac{1}{u \bullet u} uu^T \in \mathbb{R}^{n \times n},$$

where  $u^T = (u_1 \ u_2 \ \dots \ u_n) \in \mathbb{R}^{1 \times n}$ . Use this to give an expression for  $[\rho_u]$ .

**Exercise 2.** (3+3 = 6 Points) Show that for all  $u \in \mathbb{R}^n$  with  $u \neq 0$  the projection  $P_u$  and the reflection  $\rho_u$  satisfy for all  $x \in \mathbb{R}^n$  the following two properties:

- i)  $P_u(P_u(x)) = P_u(x)$ .
- ii)  $\rho_u(\rho_u(x)) = x$ .

**Exercise 3.** (6 Points) Decide if the following two linear maps are invertible. Determine their inverses if they exist.

$$F : \mathbb{R}^3 \rightarrow \mathbb{R}^3, \\ \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \mapsto \begin{pmatrix} 2x_2 + 2x_3 \\ x_1 - 4x_2 + 6x_3 \\ x_2 + x_3 \end{pmatrix},$$

$$G : \mathbb{R}^3 \rightarrow \mathbb{R}^3, \\ \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \mapsto \begin{pmatrix} 10x_1 + x_2 - 26x_3 \\ x_1 - 2x_3 \\ -x_1 + x_3 \end{pmatrix}.$$

くま先生の  
**簡単数学用語**  
**解説コーナー**



Hello ~ クマ先生 here. I hope the midterms went well for all of you.

This week, we have four words relating to the subject of linear maps:

幾何                      射影                      回転                      鏡映

These four words are: kika (**geometry**), shaei (**projection**), kaiten (**rotation**), and kyoei (**reflection**).

To describe a linear map, one must add 変換 after the word. For example, the reflection linear map is 鏡映変換 and the projection linear map is 射影変換.

Anyway, now, a breakdown of the individual (new) 漢字 that makes up these words:

幾

- Commonly read as "いく". This kanji means "(how) much" or "countless". While uncommonly used (as these words are usually written in hiragana), this kanji is used in the word 幾つ (how much) and 幾ら (how much (price)).

何

- Commonly read as "なに", this kanji means "what". This kanji is used very commonly in everyday life, one of them being 何 (meaning "what?").

影

- This kanji means "shadow". It refers to how a projection is basically an "image" (or shadow) of something when projected onto another thing. One word that includes this kanji that might be familiar is 火影 (Hokage).

回

- This kanji means "(to) turn". It refers to how rotations are.. things turning! A common use of this kanji is 回る (to turn).

転

- This kanji means "(to) turn". Again, it refers to how rotation turns things around. Common uses of this kanji are 自転車 (bicycle) and 転ぶ (to tumble).

鏡

- Commonly read as かがみ, this kanji means "mirror". It refers to how reflection, in a sense, requires a mirror (or a mirror plane). One common use of this kanji is in 眼鏡 (Spectacles).

映

- This kanji means "(to) project (light)". It refers to how reflection can be thought of as "projecting" through a mirror. The most common use of this kanji is 映画 (movie).

And that's it for today's (Mathematical) Japanese word(s). またね～