

## Homework 7: Orthogonality & Least squares

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Deadline: 30th January, 2022

**Exercise 1.** (6 Points) Let  $U = \text{span}\{u_1, u_2, u_3, u_4\} \in \mathbb{R}^4$ , where

$$u_1 = \begin{pmatrix} 3 \\ -3 \\ 5 \\ 7 \end{pmatrix}, \quad u_2 = \begin{pmatrix} 0 \\ 1 \\ 4 \\ 6 \end{pmatrix}, \quad u_3 = \begin{pmatrix} 3 \\ -2 \\ 9 \\ 13 \end{pmatrix}, \quad u_4 = \begin{pmatrix} 3 \\ -4 \\ 1 \\ 1 \end{pmatrix}.$$

- Determine a basis  $B = (b_1, \dots, b_m)$  of  $U$ .
- Calculate the coordinate vectors  $[u_j]_B \in \mathbb{R}^m$  for  $j = 1, 2, 3, 4$ .

**Exercise 2.** (6 Points) We define the following vectors

$$b_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix}, \quad b_2 = \begin{pmatrix} 1 \\ 2 \\ 0 \\ 1 \end{pmatrix}, \quad b_3 = \begin{pmatrix} 1 \\ 2 \\ 1 \\ 3 \end{pmatrix}.$$

These form a basis  $B = (b_1, b_2, b_3)$  of the subspace  $U = \text{span}\{b_1, b_2, b_3\} \subset \mathbb{R}^4$  (You do not need to show this). Use the Gram-Schmidt algorithm to construct an orthonormal basis  $F = (f_1, f_2, f_3)$  of  $U$  from  $B$ .

**Exercise 3.** (8 Points) Assume we have the following data points

$i$	1	2	3	4
$x_i$	0	1	2	3
$y_i$	2	1	3	4

- Find the line of best fit for the above data, i.e. find  $a, b \in \mathbb{R}$  such that the function  $l(x) = ax + b$  minimizes the sum of squares  $\sum_{i=1}^4 (l(x_i) - y_i)^2$ .
- Interpolate the data by a quadratic polynomial. For this find  $c, d, e \in \mathbb{R}$  such that the function  $p(x) = cx^2 + dx + e$  minimizes  $\sum_{i=1}^4 (p(x_i) - y_i)^2$ .
- Draw the data points and the graphs of  $l$  and  $p$  into one diagram.

For both (i) and (ii) solve the exercise by finding the solutions to the normal equation (Lecture 13).

くま先生の  
**簡単数学用語**  
**解説コーナー**



Happy New Year! This is the last HW for LA1, and therefore this is the last Japanese corner.

Today's words are some of the ones used in this homework!

さいしょう にじょうほう  
**最小二乗法**

せい き ちよっこう きてい  
**正規直交基底**

These words are: saishou nijouhou (**least-square method**) and seikichokkou kitei (**orthonormal basis**). Today's words (and the last ones for this semester) are:

さい  
**最**

- This kanji means "(the) **most**". It, combined with 小 will make <sup>さいしょう</sup>最小, meaning "smallest". A common use is in <sup>さいきん</sup>最近 (recently).

じょう  
**乗**

- This kanji means "**to ride**". A common example will be found when one travels and must change lines (e.g. JR or subway), namely <sup>の</sup>乗り換え (interchange).

せい  
**正**

- This kanji means "**right**". Here, <sup>せい き</sup>正規 means "normal". This kanji is used in <sup>せいぎ</sup>正義 (justice), <sup>せいかい</sup>正解 (correct answer), and <sup>しゅうせい</sup>修正 (edit).

き  
**規**

- This kanji (combined with 正) means "**conform (to)**". This kanji is a very uncommon kanji.

ちよっ  
**直**

- This kanji means "**straight**". In this context, <sup>ちよっこう</sup>直交 means "orthogonal". This kanji is used in <sup>ちよくせつ</sup>直接 (upfront) and <sup>しやうじき</sup>正直 (honestly).

こう  
**交**

- This kanji means "**swap**". This kanji is used in <sup>こうさてん</sup>交差点 (crossroads) and <sup>こうたい</sup>交代 (to exchange).

And that's it for today. I wish you luck in the finals. Thank you for reading this Japanese corner.

