

Homework 4: Linear maps II

Deadline: 28th November, 2021

Exercise 1. (4 Points) Let $F : \mathbb{R}^n \rightarrow \mathbb{R}^m$ be a linear map. Show that the following two statements are equivalent:

- i) F is injective.
 - ii) The only solution to $F(x) = 0$ is $x = 0$.
- (i.e. show that i) implies ii) and ii) implies i)).

Exercise 2. (3+5 = 8 Points) Let $u = \begin{pmatrix} u_1 \\ \vdots \\ u_n \end{pmatrix} \in \mathbb{R}^n$ be with $u \neq 0$.

- i) Show that the reflection $\rho_u : \mathbb{R}^n \rightarrow \mathbb{R}^n$ is a linear map.
- ii) Show that the matrix of the projection $P_u : \mathbb{R}^n \rightarrow \mathbb{R}^n$ is given by

$$[P_u] = \frac{1}{u \bullet u} uu^T \in \mathbb{R}^{n \times n},$$

where $u^T = (u_1 \ u_2 \ \dots \ u_n) \in \mathbb{R}^{1 \times n}$. Use this to give an expression for $[\rho_u]$.

Exercise 3. (3+3 = 6 Points) Show that for all $u \in \mathbb{R}^n$ with $u \neq 0$ the projection P_u and the reflection ρ_u satisfy for all $x \in \mathbb{R}^n$ the following two properties:

- i) $P_u(P_u(x)) = P_u(x)$.
- ii) $\rho_u(\rho_u(x)) = x$.

くま先生の
簡単数学用語
解説コーナー



Hello ~ クマ先生 here. I hope the midterms went well for all of you.

This week, we have four words relating to the subject of linear maps:

幾何 射影 回転 鏡映

These four words are: kika (**geometry**), shaei (**projection**), kaiten (**rotation**), and kyoei (**reflection**).

To describe a linear map, one must add ^{へんかん}変換 after the word. For example, the reflection linear map is ^{きょうえいへんかん}鏡映変換 and the projection linear map is ^{しゃえいへんかん}射影変換.

Anyway, now, a breakdown of the individual (new) ^{かんじ}漢字 that makes up these words:

幾

- Commonly read as "いく". This kanji means "**(how) much**" or "**countless**". While uncommonly used (as these words are usually written in hiragana), this kanji is used in the word 幾つ (how much) and 幾ら (how much (price)).

何

- Commonly read as "なに", this kanji means "**what**". This kanji is used very commonly in everyday life, one of them being 何 (meaning "what?").

影

- This kanji means "**shadow**". It refers to how a projection is basically an "image" (or shadow) of something when projected onto another thing. One word that includes this kanji that might be familiar is ^{ほかげ}火影 (Hokage).

回

- This kanji means "**(to) turn**". It refers to how rotations are.. things turning! A common use of this kanji is ^{まわ}回る (to turn).

転

- This kanji means "**(to) turn**". Again, it refers to how rotation turns things around. Common uses of this kanji are ^{じてんしゃ}自転車 (bicycle) and ^{ころ}転ぶ (to tumble).

鏡

- Commonly read as ^{かがみ}かがみ, this kanji means "**mirror**". It refers to how reflection, in a sense, requires a mirror (or a mirror plane). One common use of this kanji is in ^{めがね}眼鏡 (Spectacles).

映

- This kanji means "**(to) project (light)**". It refers to how reflection can be thought of as "projecting" through a mirror. The most common use of this kanji is ^{えいが}映画 (movie).

And that's it for today's (Mathematical) Japanese word(s). またね ~