Homework 2: Matrices, vectors & the rank of a matrix

Deadline: 31th October, 2021

Exercise 1. (3+3 = 6 Points) Show that for all $A \in \mathbb{R}^{m\times n}$, $x, y \in \mathbb{R}^n$ and $\lambda \in \mathbb{R}$ we have

i) $A(x+y) = Ax + Ay$,

ii) $A(\lambda x) = \lambda(Ax)$.

(Without using Proposition 2.4. from the lecture).

Exercise 2. (4 Points) Let $p(x) = a_0 + a_1x + a_2x^2 + a_3x^3$ be a polynomial of degree 3 with real coefficients $a_0, a_1, a_2, a_3 \in \mathbb{R}$. For this polynomial p we define the vector v_p by

$$v_p = \begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{pmatrix} \in \mathbb{R}^4 .$$

Find a matrix $D \in \mathbb{R}^{4\times 4}$, such that $v_{p'} = Dv_p$, where p' denotes the derivative of the polynomial p with respect to x. What is the rank of D?

Exercise 3. (4+3+1 = 8 Points) Let $a, b, c, d \in \mathbb{R}$ and $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$.

i) Show that $\mathrm{rk}(A) = 2$ if and only if $ad - bc \neq 0$.

ii) What can you say about a, b, c, d if $\mathrm{rk}(A) = 1$? Consider the following subset of $\mathbb{R}^2$

$$L = \{x \in \mathbb{R}^2 \mid x = Av \text{ for some } v \in \mathbb{R}^2\}.$$

How does L look like if $\mathrm{rk}(A) = 1$? How does it look like if $\mathrm{rk}(A) = 2$?

iii) What can you say about a, b, c, d if $\mathrm{rk}(A) = 0$?

Version: October 15, 2021

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くま先生の 簡単数学用語 解説コーナー

Hello ～ クマ先生 here. With another homework comes another Japanese lesson～

This week, I have three words prepared for this time:

行列 (ぎょうれつ)　　方程式 (ほうていしき)　　ベクトル

These three words are: gyouretsu (**matrix**), houteishiki (**equation**), and bekutoru (**vector**). While the Japanese word for vector is a direct transliteration of the English word vector, the other two are not so.

The first word (行列) also has a meaning in everyday life: a procession (e.g. a wedding procession).

The second word (方程式), however, appears more in the sciences. Famous examples include Schrödinger's Equation (シュレディンガー方程式) and Maxwell's Equations (マクスウェルの方程式). Generally, however, this word can be used to describe any type of equation. For example, 行列方程式 (Matrix Equation), 代数方程式 (Algebraic Equation), ベクトル方程式 (Vector Equation) and 線型方程式 (Linear Equation).

Anyway, now, a breakdown of the individual 漢字 that makes up these two words:

行 (ぎょう) - This kanji means "**row(s)**". It refers to how matrices have "rows". In fact, the "row" of a matrix is called a 行 in Japanese. In everyday life, this kanji very common, as it is the kanji used in 行きます (meaning "to go" in Japanese).

列 (れつ) - This kanji means "**column(s)**". As such, the Japanese word for matrix literally means "rows of columns" or "columns of rows", which is an apt desrciption of what a matrix is. In everyday life, this kanji shows up because of its other meaning : a queue (i.e. a queue in front of a shop).

方 (ほう) - This kanji means "**direction**" or "**way**". It refers to how an equation gives direction(s) on how the variables are related to one another. This kanji appears in a few words in everyday life, including 方法 (method), (あの) 方 (that person), and 貴方 (you; kanji literally means "precious person").

程 (てい) - This kanji means "**about**". In a sense, 方程式 (Equation) tells us something about some natural phenomenon. This kanji is uncommon in everyday life.

式 (しき) - This kanji means "**formula**" or "**ceremony**". This kanji can be found in any ceremony in everyday life, such as 卒業式 (Graduation Ceremony), 入学式 (Entrance ceremony), and 結婚式 (Wedding ceremony), among others.

And that's it for today's (Mathematical) Japanese word(s). またね～