# Homework 1: Linear systems

Deadline: 17th October, 2021

**Exercise 1.** (2 Points) Try to solve the exercises below and write the solutions down by hand (paper, tablet) <u>or</u> by computer (Latex only. No word!). Write your name, the homework number and the course name on the first page of your solution. Create **one pdf-file** (for example, by using a scanner app on your phone) and submit it before the deadline ends in NUCT at the Assignment "Homework 1". Use exactly the following format as a filename: "**Familyname_Givenname_LA1_HW1.pdf**".

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A linear system is said to be on **row-reduced echelon form** if the following three conditions are satisfied:

(i) The first (that is, the leftmost) variable in each equation has coefficient 1.

(ii) If $x_i$ is the first variable in one of the equations, then it does not occur in any other equation in the system.

(iii) If $x_i$ is the first variable in one equation, then the equations below it do not contain any of the variables $x_1, x_2, \ldots, x_{i-1}$.

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**Exercise 2.** (2+2+2+2 = 8 Points) Which of the following linear systems are on row-reduced echelon form? For those that are not, find an equivalent system (i.e. one which has the same solutions) that is on row-reduced echelon form. For each system, find all solutions.

i) $\begin{cases} x_1 & - & 2x_2 & + & x_3 & = & 7 \\ 3x_1 & & & + & 2x_3 & = & 1 \\ -2x_1 & + & x_2 & + & 4x_3 & = & 7 \end{cases}$

ii) $x_1 + 2x_2 + 3x_3 + 4x_4 = 5$

iii) $\begin{cases} x_1 & + & x_2 & + & x_3 & + & 2x_4 & = & 0 \\ & & x_2 & & & + & x_4 & = & 0 \end{cases}$

iv) $\begin{cases} x_1 & + & 2x_2 & = & 3 \\ 4x_1 & + & 8x_2 & = & 16 \end{cases}$

**Exercise 3.** (8 Points) A Japanese restaurant in 八事 (Yagoto, a neighbourhood in Nagoya) is holding an Ebi Festival, and thus is only selling three types of dishes: Ebi Sushi (¥370), Ebi Tempura Don (¥590), and Ebi Fry Bentō (¥830). One serving of Ebi Sushi requires 3 ounces of shrimp, 1 cup of rice, and 3 tablespoon of shouyu. 5 ounces of shrimp, 4 cups of rice, and $\frac{5}{2}$ tablespoons of shouyu are needed for one portion of Ebi Tempura Don. For one serving of Ebi Fry Bento, 8 ounces of shrimp, 3 cups of rice, and $\frac{1}{2}$ tablespoons of shouyu are needed. In one certain day, the store expended 1000 ounces of shrimp, 500 cups of rice, and 500 tablespoons of shouyu.

The market prices are: ¥50 per ounce of shrimp, ¥30 per cup of rice, and ¥5 per tablespoon of shouyu. Given all these information, how much profit did the restaurant make on this certain day? Describe this problem by using a linear system, bring the linear system on row-reduced echelon form and solve it.

**Exercise 4.** (6 Points) Decide for which real numbers $a \in \mathbb{R}$ the following linear system has solutions. Give all the solutions in these cases.

$$\begin{cases} (a-1)^2 x_1 & + & x_2 & + & ax_3 & = & 0 \\ x_1 & + & x_2 & & & = & 0 \\ 2x_1 & + & 3x_2 & + & x_3 & = & a \end{cases}.$$

くま先生の
簡単数学用語
解説コーナー

Welcome to Nagoya University's Linear Algebra I Course ! My name is クマ先生 (せんせい), and I'm here to teach you some mathematical Japanese words that will probably (not) be helpful to you guys while living here.

Now let's get down to business. Today's word is..

### せんけいだいすうがく
# 線形代数学
senkei daisuugaku

This word literally means **linear algebra**. Now you know how to say this course's name in Japanese～

And then you can say this sentence in your next Japanese class:

### わたし せんけいだいすうがく だいす
# 私 は線形代数学が大好きです。

Anyway, now, a breakdown of the individual 漢字 (かんじ) (kanji) that makes up this word:

| | | |
|---|---|---|
| せん<br>線 | - | This kanji means "**line**". It refers to the fact that this branch of algebra is interested with "linear" functions. In everyday life, this kanji can be found in subway line names (examples in Nagoya: 名城線 (めいじょうせん) and 東山線 (ひがしやません)) |
| けい<br>形 | - | This kanji means "form". If written after another word, the overall word has the meaning "**similar to** (word before it)". In this case, 線形 (せんけい) means "similar to line", or "**linear**". A common (everyday) example is 人形 (にんぎょう) (ningyō), which means "doll", which are traditionally made in the likeness of humans. |
| だい<br>代 | - | This kanji means "**change**". In the sense that it is used, it refers to how algebra 代数学 (だいすうがく)) "changes" numbers to letters and/or other symbols. A common use of this kanji is in 時代 (じだい) (jidai), which means era, a "change" of the times. |
| すう<br>数 | - | This kanji means "**number**". In a sense, 数学 (すうがく) (mathematics) is a study of "numbers". This character usually appears when numbers are involved. In fact, the word "digit" in Japanese is 数字 (すうじ), literally meaning "number word". |
| がく<br>学 | - | This kanji means "**study**". This kanji is very common, used in a large number of everyday words, such as 学校 (がっこう) (school), 大学 (だいがく) (university), 科学 (かがく) (science), 学生 (がくせい) (student), and 留学 (りゅうがく) (studying abroad). |

Now we see that the Japanese word for linear algebra is an apt description of the field of study.

And that's it for today's (Mathematical) Japanese word. またね ～