

Question 1

Consider the following linear system:

$$x_1 + x_2 - 8x_3 = 6$$

$$x_2 + 2x_3 = 2$$

(A) This linear system is on rref.

(B) This linear system has infinitely many solutions.

(C) $x_1 = 14$
 $x_2 = 0$
 $x_3 = 1$ is a solution.

(D) There are 3 pivot variables.

Question 2

We define the matrix A and vector b by

$$A = \begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix}, \quad b = \begin{pmatrix} 2 \\ 6 \end{pmatrix}.$$

(A) $\text{rk}(A) < \text{rk}(A|b)$

(B) $\text{rref}(A) = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$

(C) There exists a $x \in \mathbb{R}^2$ with $Ax = b$.

(D) There exists a $x \in \mathbb{R}^2$ with $Ab = x$.

Question 3 Let $A \in \mathbb{R}^{m \times n}$ and $b \in \mathbb{R}^m$ be arbitrary, $(m, n \geq 1)$

Which statements are true for any choices of A and b ?

(A) $\text{rk}(A) \leq m$

(B) $\text{rk}(A) \leq n$

(C) $\text{rk}(A) \geq 1$

(D) $\text{rk}(A) \leq \text{rk}(A|b)$

