Homework 6: Subspaces

Deadline: 23th December, 2020

Exercise 1. (2+4=6 Points) Show the following without using Proposition 8.5:

i) For $v_1, \dots, v_n \in \mathbb{R}^m$ the set $\operatorname{span}\{v_1, \dots, v_n\}$ is a subspace of $\mathbb{R}^m$.

ii) If $U \subset \mathbb{R}^m$ is a subspace and $v_1, \dots, v_n \in U$ then $\operatorname{span}\{v_1, \dots, v_n\} \subset U$.

Exercise 2. (6+4=10 Points)

i) Which of the following subsets are subspaces? Justify your answers.

$$
\begin{aligned}
U_1 &= \left\{ x \in \mathbb{R}^3 \mid \|x\| \leq 1 \right\} , \\
U_2 &= \left\{ x \in \mathbb{R}^3 \mid 2x_1 - x_2 = x_3 \right\} , \\
U_3 &= \left\{ x \in \mathbb{R}^3 \mid x_1^2 - x_1 = 0 \right\} , \\
U_4 &= \left\{ x \in \mathbb{R}^4 \mid P_u(x) = 0 \right\} , \quad \text{where } u = \begin{pmatrix} 1 \\ 0 \\ 2 \\ -1 \end{pmatrix} .
\end{aligned}
$$

ii) For the subspaces U in i): Find vectors $v_1, \dots, v_l$ such that $U = \operatorname{span}\{v_1, \dots, v_l\}$.

(No-points-challenge for ii): Try to choose the $v_1, \dots, v_l$ such that they are pairwise orthogonal and all of them have norm 1. You will learn later in the semester why this is cool)

Exercise 3. (2+2=4 Points) Find an example of a subset $U \subset \mathbb{R}^2$ which is <u>not</u> a subspace, but which

i) includes 0 and which is closed under addition.

ii) includes 0 and which is closed under scalar multiplication.

In other words: Find examples of subsets, which just satisfy 2 of the 3 conditions for subspaces.