

Homework 5: Invertible linear maps & matrices

Deadline: 6th December, 2020

Exercise 1. (4+4=8 Points)

- i) Decide if the following two linear maps are invertible. Determine their inverses if they exist.

$$F : \mathbb{R}^3 \longrightarrow \mathbb{R}^3, \quad G : \mathbb{R}^3 \longrightarrow \mathbb{R}^3,$$
$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \longmapsto \begin{pmatrix} -x_1 + x_2 + 5x_3 \\ 2x_1 - x_2 + 2x_3 \\ -x_1 + x_2 + 4x_3 \end{pmatrix}, \quad \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \longmapsto \begin{pmatrix} x_1 + x_2 + 2x_3 \\ 2x_1 + 2x_2 + 4x_3 \\ 2x_1 + 3x_2 + 5x_3 \end{pmatrix}.$$

- ii) The **kernel** of a linear map $H : \mathbb{R}^n \rightarrow \mathbb{R}^m$ is defined by

$$\ker(H) = \{x \in \mathbb{R}^n \mid H(x) = 0\}.$$

Determine $\ker(F)$ and $\ker(G)$.

Exercise 2. (2+2 = 4 Bonus points)

- i) Suppose you fill the entries of a 2×2 matrix randomly with 0 or 1 by tossing a coin for each entry. What is the probability that this matrix is invertible?
- ii) Do the same as in i), but now fill the matrix randomly with -1 and 1 . What is the probability that this matrix is invertible?