Homework 2: Matrices & Vectors

Deadline: 25th October, 2020

Exercise 1. (2+2 = 4 Points) Show that for all $A \in \mathbb{R}^{m\times n}$, $x, y \in \mathbb{R}^n$ and $\lambda \in \mathbb{R}$ we have

i) $A(x+y) = Ax + Ay$,

ii) $A(\lambda x) = \lambda(Ax)$.

(Without using Proposition 2.4. from the lecture).

Exercise 2. (3+3 = 6 Points) We define the following matrices and vectors:

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad C = \begin{pmatrix} -1 & 0 \\ 2 & 4 \\ 0 & -3 \end{pmatrix}, \quad D = \begin{pmatrix} 0 & 8 & 0 \\ 1 & 2 & -1 \end{pmatrix}, \quad E = \begin{pmatrix} \frac{3}{5} & -\frac{4}{5} \\ \frac{4}{5} & \frac{3}{5} \end{pmatrix},$$

$$t = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \quad u = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}, \quad v = \begin{pmatrix} 3 \\ -4 \end{pmatrix}, \quad w = \begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix}.$$

i) Decide which of the following expressions are defined. Evaluate them if possible.

$$At, \quad Au, \quad wA, \quad 2A, \quad A+B, \quad A+C, \quad A+D, \quad \frac{3}{4}Bt, \quad Bu, \quad B+B, \quad Dw,$$
$$Cv, \quad t+u, \quad tu, \quad -v, \quad u+w, \quad t-u, \quad \frac{1}{2}w, \quad C+w, \quad Et, \quad Ev \quad E\,(Ev)\,.$$

ii) Draw the following vectors in $\mathbb{R}^2$

$$t, \quad ,v, \quad -2t, \quad t-\frac{1}{2}v, \quad v+t, \quad t+v, \quad Et, \quad Ev, \quad E\,(Ev)\,, \quad Bt, \quad Bv\,.$$

Can you guess what happens in general to a vector in $\mathbb{R}^2$ when you multiply it with B or E? Try to give a geometric interpretation. (without a proof)

Exercise 3. (2+4 = 6 Points)

i) Give examples of matrices $A, B, C \in \mathbb{R}^{3\times 3}$, which are all <u>not</u> on row-reduced echelon form, such that $\mathrm{rk}(A) = 1$, $\mathrm{rk}(B) = 2$, $\mathrm{rk}(C) = 3$.

ii) Let $a, b, c, d \in \mathbb{R}$ with $ad - bc \neq 0$ and $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$. Show that $\mathrm{rk}(A) = 2$.

Version: October 10, 2020