

## Homework 1: Linear systems

Deadline: 18th October, 2020

**Exercise 1.** (1 Point) Solve the Exercises 2 & 3 and write the solutions down by hand (paper, tablet) or by computer (Latex). Create one pdf-file (for example, by using a scanner app on your phone) and send it before the deadline ends (any time on 18th October is fine) to [henrik.bachmann@math.nagoya-u.ac.jp](mailto:henrik.bachmann@math.nagoya-u.ac.jp). Use the following format as a filename: "Familyname.Firstname.LA1.HW1.pdf".

A linear system is said to be on **row-reduced echelon form** if the following three conditions are satisfied:

- (i) The first (that is, the leftmost) variable in each equation has coefficient 1.
- (ii) If  $x_i$  is the first variable in one of the equations, then it does not occur in any other equation in the system.
- (iii) If  $x_i$  is the first variable in one equation, then the equations below it do not contain any of the variables  $x_1, x_2, \dots, x_{i-1}$ .

**Exercise 2.** (2+2+2+1+1 = 8 Points) Which of the following linear systems are on row-reduced echelon form? For those that are not, find an equivalent system (i.e. one which has the same solutions) that is on row-reduced echelon form. For each system, find all solutions.

i) 
$$\begin{cases} x_1 + x_2 + x_3 + 2x_4 = 0 \\ \phantom{x_1} + x_2 \phantom{+ x_3} - x_4 = 0 \end{cases}$$

ii) 
$$\begin{cases} x_1 + 4x_2 + 7x_3 = 1 \\ 2x_1 + 5x_2 + 8x_3 = 2 \\ 3x_1 + 6x_2 + 10x_3 = 1 \end{cases}$$

iii)  $x_1 + 2x_2 + 3x_3 + 4x_4 = 5$

iv) 
$$\begin{cases} x_1 = 2 \\ x_2 = 0 \\ x_3 = 2 \end{cases}$$

v) 
$$\begin{cases} x_1 + 3x_2 = 1 \\ 3x_1 + 9x_2 = 2 \end{cases}$$

**Exercise 3.** (5 Points) Decide for which real numbers  $a \in \mathbb{R}$  the following linear system has solutions. Give all the solutions in these cases.

$$\begin{cases} 2x_1 + 12x_2 + 7x_3 = 12a + 7 \\ 2x_1 + 4x_2 + 2x_3 = 12a \\ x_1 + 10x_2 + 6x_3 = 7a + 8 \end{cases} .$$