Tutorial 9: Review for the final exam

Exercise 1. Let $A = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 1 & 0 \\ 2 & -1 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & 1 & 2 & 3 \\ 1 & 3 & 0 & 2 \\ 2 & 1 & -5 & -1 \end{pmatrix}$.

i) Compute the products AB and BA, or explain why they are not defined.

ii) Determine whether or not the matrices A and B are invertible and, if they are, compute their inverses.

iii) Find a matrix C, such that $AC = B$.

Exercise 2.

i) Give an example of a subspace U of $\mathbb{R}^3$ with $\dim U = 2$.

ii) Give an example of three vectors $u_1, u_2, u_3 \in U$, such that

$$U = \operatorname{span}\{u_1, u_2\} = \operatorname{span}\{u_1, u_3\} = \operatorname{span}\{u_2, u_3\}.$$

iii) Determine a basis for $U^\perp$.

iv) Find a $w \in \mathbb{R}^3$ such that $\operatorname{span}\{u_1, u_2, w\} = \mathbb{R}^3$, where (u_1, u_2) is a basis of U.

v) Find linear maps $H : \mathbb{R}^3 \to \mathbb{R}^3$ and $G : \mathbb{R}^3 \to \mathbb{R}^3$, with $\ker H = U$ and $\operatorname{im} G = U$.

(in ii)-v) the U is the subspace you choose in i))

Exercise 3. Give an example of a subset $U \subset \mathbb{R}^2$, which is not a subspace, but which satisfies two of the three conditions in the definition of a subspace.

Exercise 4. We define the following linear map

$$F : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$$
$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \longmapsto \begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & -5 \\ 1 & 0 & -2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}.$$

i) Determine a basis $B = (b_1, \dots, b_m)$ for $\operatorname{im}(F)$.

ii) Check for which $t \in \mathbb{R}$ the vector $x = \begin{pmatrix} 3 \\ 1 \\ t \end{pmatrix}$ is an element in $\operatorname{im}(F)$. Determine the coordinate vector $[x]_B$, where B is the basis you calculated in i).

iii) Calculate an orthonormal basis for $\operatorname{im}(F)$.

Exercise 5.

i) Give an example of a matrix $A \in \mathbb{R}^{3\times 2}$ and $b \in \mathbb{R}^3$ such that $Ax = b$ has no solutions.

ii) Find a $x \in \mathbb{R}^2$ such that $\|Ax - b\|$ is minimal, where A and b are your choices in i).

Exercise 6.

i) Exchange and discuss your examples of Exercise 2, 3 and 5 with your fellow students.

ii) Recall Homework 4: Ex. 1; Homework 5: Ex. 1, 2; Homework 6: Ex. 1,2,3; Quiz 3,5,6.

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