

Tutorial 7: Kernel, Image and Basis

Exercise 1. Consider the following linear map

$$F : \mathbb{R}^4 \longrightarrow \mathbb{R}^4$$
$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} \longmapsto \begin{pmatrix} 1 & 2 & 1 & -1 \\ 2 & 4 & 2 & -2 \\ 3 & 6 & 5 & 1 \\ 4 & 8 & 3 & -6 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}$$

and the two vectors

$$v = \begin{pmatrix} 1 \\ 2 \\ 11 \\ 0 \end{pmatrix}, \quad w = \begin{pmatrix} 1 \\ -2 \\ 2 \\ -1 \end{pmatrix}.$$

- i) Check if v and w belong to $\ker(F)$ and/or $\text{im}(F)$.
- ii) Find a basis for $\ker(F)$.
- iii) Find a basis for $\text{im}(F)$.

Exercise 2. Determine a basis of the following subspace

$$U = \text{span} \left\{ \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 6 \\ -1 \end{pmatrix} \right\}.$$