Tutorial 5: Functions & Linear maps

For a function $f : X \to Y$ the **image of** f is defined by

$$\operatorname{im}(f) = f(X) = \{y \in Y \mid \exists x \in X : y = f(x)\} \subset Y\,.$$

A function $f : X \to Y$ is called

i) **injective** if $f(x_1) \neq f(x_2)$ whenever $x_1 \neq x_2$. $(x_1, x_2 \in X)$

ii) **surjective** if $\operatorname{im}(f) = Y$.

iii) **bijective** if it is injective and surjective.

Exercise 1. For $X = \mathbb{R}$ try to find an example of a function $f : X \to X$, which is

i) not injective and not surjective.

ii) injective but not surjective.

iii) not injective but surjective.

iv) bijective.

Calculate the image of f in each case. Do the same for $X = \{1, 2, 3\}$. Is it possible to find examples for all cases?

Exercise 2. We define for a matrix $A \in \mathbb{R}^{m\times n}$ the linear map

$$\begin{aligned} F_A : \mathbb{R}^n &\longrightarrow \mathbb{R}^m \\ x &\longmapsto Ax\,. \end{aligned}$$

Try to find examples of numbers m, n and matrices $A \in \mathbb{R}^{m\times n}$, such that F_A is

i) not injective and not surjective.

ii) injective but not surjective.

iii) not injective but surjective.

iv) bijective.

Calculate the image of F_A in each case.

Version: November 2, 2019