Homework 4: Inverses & Subspaces & Bases

Deadline: 17th December, 2019

Exercise 1. (6 Points)

i) Decide if the following two linear maps are invertible. Determine their inverses if they exist.

$$F : \mathbb{R}^3 \longrightarrow \mathbb{R}^3\,, \quad \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \longmapsto \begin{pmatrix} 2x_2 + 2x_3 \\ x_1 - 4x_2 + 6x_3 \\ x_2 + x_3 \end{pmatrix}\,, \qquad G : \mathbb{R}^3 \longrightarrow \mathbb{R}^3\,, \quad \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \longmapsto \begin{pmatrix} 10x_1 + x_2 - 26x_3 \\ x_1 - 2x_3 \\ -x_1 + x_3 \end{pmatrix}\,.$$

ii) Determine $\ker(F)$ and $\ker(G)$.

Exercise 2. (6 Points) Which of the following subsets are subspaces? Justify your answers.

$$\begin{aligned}
U_1 &= \left\{x \in \mathbb{R}^3 \mid x_1 + x_2 + x_3 = 0\right\}\,,\\
U_2 &= \left\{x \in \mathbb{R}^3 \mid x_1 \cdot x_2 \cdot x_3 = 0\right\}\,,\\
U_3 &= \{x \in \mathbb{R}^n \mid Ax = Bx\}\,, \quad \text{where } A, B \in \mathbb{R}^{m\times n},\\
U_4 &= \left\{x \in \mathbb{R}^2 \mid x_1 \leq x_2\right\}\,,\\
U_5 &= \{x \in \mathbb{R}^n \mid x \bullet u = 0\}\,, \quad \text{where } u \in \mathbb{R}^n.
\end{aligned}$$

Exercise 3. (6 Points)

i) Let $U \subset \mathbb{R}^m$ be a subspace and $v_1, \dots, v_n \in U$. Show that $\operatorname{span}\{v_1, \dots, v_n\} \subset U$.

ii) Let $U, V \subset \mathbb{R}^m$ be subspaces. Decide whether the union $U \cup V$ is also a subspace or not.

iii) Let $U, V \subset \mathbb{R}^m$ be subspaces. Decide whether the intersection $U \cap V$ is also a subspace or not.

Exercise 4. (4 Points) Determine a basis of the following subspace

$$U = \operatorname{span}\left\{ \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 3 \\ 3 \\ 8 \end{pmatrix}, \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} \right\}.$$

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