Tutorial 1b (Linear Algebra)

Homework 1 (Solution)

Exercise 1

i) $\begin{cases} x_1 + x_2 + x_3 + 2x_4 = 0 \\ x_2 + x_4 = 0 \end{cases}$ (−1)

This linear system is not in row-reduced echelon form

$\Leftrightarrow \begin{cases} x_1 + x_3 + x_4 = 0 \\ x_2 + x_4 = 0 \end{cases}$ ← row-reduced echelon form

Set $x_3 = t_1$, $x_4 = t_2$ for $t_1, t_2 \in \mathbb{R}$.

Solution : $\begin{aligned} x_1 &= -t_1 - t_2 \\ x_2 &= -t_2 \\ x_3 &= t_1 \\ x_4 &= t_2 \end{aligned}$

ii) (−3) (−2) $\begin{cases} x_1 + 4x_2 + 7x_3 = 1 \\ 2x_1 + 5x_2 + 8x_3 = 2 \\ 3x_1 + 6x_2 + 10x_3 = 1 \end{cases} \Leftrightarrow \left(-\frac{1}{3}\right) \begin{cases} x_1 + 4x_2 + 7x_3 = 1 \\ -3x_2 - 6x_3 = 0 \\ -6x_2 - 11x_3 = -2 \end{cases}$

$\Leftrightarrow$ (6) $\begin{cases} x_1 + 4x_2 + 7x_3 = 1 \\ x_2 + 2x_3 = 0 \\ -6x_2 - 11x_3 = -2 \end{cases} \Leftrightarrow$ (−7) (−2) $\begin{cases} x_1 + 4x_2 + 7x_3 = 1 \\ x_2 + 2x_3 = 0 \\ x_3 = -2 \end{cases}$

$\Leftrightarrow$ (−4) $\begin{cases} x_1 + 4x_2 = 15 \\ x_2 = +4 \\ x_3 = -2 \end{cases} \Leftrightarrow \begin{cases} x_1 = -1 \\ x_2 = 4 \\ x_3 = -2 \end{cases}$.

iii) The linear system

$$x_1 + 2x_2 + 3x_3 + 4x_4 = 5$$

is already on row-reduced echelon form. Set $x_2 = t_1, x_3 = t_2$

$x_4 = t_3$ for $t_1, t_2, t_3 \in \mathbb{R}$.

$$\text{Solution: } \begin{cases} x_1 = 5 - 2t_1 - 3t_2 - 4t_3 \\ x_2 = t_1 \\ x_3 = t_2 \\ x_4 = t_3 \end{cases}$$

iv) $$\begin{cases} x_1 + 2x_2 = 3 \\ 4x_1 + 8x_2 = 16 \end{cases} \iff \begin{cases} x_1 + 2x_2 = 3 \\ 0 = 4 \end{cases}$$

(-4)

This linear system han no solution, since $0 \neq 4$.

v) Already on row-reduced echelon form with solution:

$$x_1 = 6$$
$$x_2 = 9$$
$$x_3 = 1$$

Exercise 2

$$\begin{cases} x_1 + x_2 = 2 \\ ax_1 + 2x_2 = b \end{cases} \iff \begin{cases} x_1 + x_2 = 2 \\ (2-a)x_2 = b - 2a \end{cases}$$

(-a)

Case $a \neq 2$: In this case we can divide by $2-a$ and get the solution $x_2 = \frac{b-2a}{2-a}$ and $x_1 = 2 - \frac{b-2a}{2-a} = \frac{4-b}{2-a}$.

case $a = 2$: In this case the linear system is

$$\begin{cases} x_1 + x_2 = 2 \\ 0 = b - 2a \end{cases}$$

If $b = 2a$, then we have the solution $x_1 = 2 - t$ for $t \in \mathbb{R}$.
$x_2 = t$

If $b \neq 2a$ we have no solution.

Exercise 3 We first calculate the number of portions sold on this day for each type of ramen.

M : Number of Miso ramen

T : Number of Taiwan ramen

K : Number of Tonkotsu ramen.

We get the following linear system

$$\begin{array}{l} \text{salt} \to \\ \text{garlic} \to \\ \text{chili} \to \end{array} \begin{cases} 3M + 2T + 2K = 142 \\ M + 2T + 3K = 146 \\ 0 + 4T + K = 152 \end{cases} \Leftrightarrow \begin{cases} M + 2T + 3K = 146 \\ 3M + 2T + 2K = 142 \\ 4T + K = 152 \end{cases}$$

$$\Leftrightarrow \begin{cases} M + 2T + 3K = 146 \\ -4T - 7K = -296 \\ 4T + K = 152 \end{cases} \Leftrightarrow \begin{cases} M + 2T + 3K = 146 \\ -4T - 7K = -296 \\ -6K = -144 \end{cases}$$

$$\Leftrightarrow \begin{cases} M + 2T + 3K = 146 \\ -4T - 7K = -296 \\ K = 24 \end{cases} \Leftrightarrow \begin{cases} M + 2T = 74 \\ -4T = -128 \\ K = 24 \end{cases}$$

$$\Leftrightarrow \begin{cases} M + 2T = 74 \\ T = 32 \\ K = 24 \end{cases} \Leftrightarrow \begin{cases} M = 10 \\ T = 32 \\ K = 24 \end{cases}$$

The store therefore earned $10 \cdot 700 + 32 \cdot 800 + 24 \cdot 850 = \underline{\underline{53000\,¥}}$