

Homework 1: Linear systems

Deadline: 15th October, 2019

A linear system is said to be on **row-reduced echelon form** if the following three conditions are satisfied:

- (i) The first (that is, the leftmost) variable in each equation has coefficient 1.
- (ii) If x_i is the first variable in one of the equations, then it does not occur in any other equation in the system.
- (iii) If x_i is the first variable in one equation, then the equations below it do not contain any of the variables x_1, x_2, \dots, x_{i-1} .

Exercise 1. ($2+2+1+2+1 = 8$ Points) Which of the following linear systems are on row-reduced echelon form? For those that are not, find an equivalent system (i.e. one which has the same solutions) that is on row-reduced echelon form. For each system, find all solutions.

i)
$$\begin{cases} x_1 + x_2 + x_3 + 2x_4 = 0 \\ + x_2 + x_4 = 0 \end{cases}$$

ii)
$$\begin{cases} x_1 + 4x_2 + 7x_3 = 1 \\ 2x_1 + 5x_2 + 8x_3 = 2 \\ 3x_1 + 6x_2 + 10x_3 = 1 \end{cases}$$

iii) $x_1 + 2x_2 + 3x_3 + 4x_4 = 5$

iv)
$$\begin{cases} x_1 + 2x_2 = 3 \\ 4x_1 + 8x_2 = 16 \end{cases}$$

v)
$$\begin{cases} x_1 = 6 \\ x_2 = 9 \\ x_3 = 1 \end{cases}$$

Exercise 2. (4 Points) Let $a, b \in \mathbb{R}$ be two arbitrary real numbers. Consider the following linear system:

$$\begin{cases} x_1 + x_2 = 2 \\ ax_1 + 2x_2 = b \end{cases}$$

Find all the solutions to this linear system depending on a and b .
(Hint: You need to consider different special cases of a and b separately)

Exercise 3. (4 Points) A ramen store in Sakae offers three types of ramen: Miso ramen (price for one portion: 700¥), Taiwan ramen (800¥), and Tonkotsu ramen (850¥). For one portion of Miso ramen one needs 3 tablespoons (tbsp) of salt, one clove of garlic and no chili. One portion of Taiwan ramen needs 2 tbsp. of salt, 2 cloves of garlic and 4 tbsp. of chili. For one portion of Tonkotsu ramen 2 tbsp. of salt, 3 cloves of garlic and one tbsp. of chili is needed.¹ In one day the store uses 142 tbsp. of salt, 146 cloves of garlic, and 152 tbsp. of chili.

How much money (in ¥) did the store earn on this day? Describe this problem by using a linear system and then solve it.

¹These amounts are made up and should probably not be used to make tasty ramen.