

Homework 5

Deadline: 6th February (23:55 JST), 2022

Exercise 15. Let ζ be a primitive n -th root of unity for $n \geq 3$. Show that the numbers $\epsilon = \frac{1-\zeta^k}{1-\zeta}$ for $(k, n) = 1$ are units in the ring of integers of the field $\mathbb{Q}(\zeta)$.

Exercise 16. Calculate the fundamental unit of $K = \mathbb{Q}(\sqrt{d})$ for the cases $d = 2, 5, 10$.

Exercise 17. In $\mathbb{Z}[\sqrt[4]{2}]$, show $u = 1 + \sqrt[4]{2}$ and $v = 1 + \sqrt{2}$ are units and they are multiplicatively independent: if $u^a v^b = 1$ for $a, b \in \mathbb{Z}$ then $a = 0$ and $b = 0$.