

Homework 3

Deadline: 9th January (23:55 JST), 2022

Exercise 12. Show that there are no nonzero solutions $x, y, z \in \mathbb{Z}$ for

$$x^4 + y^4 = z^2.$$

(Hint: Use Exercise 4)

Exercise 13. Let $K = \mathbb{Q}(\sqrt{d})$. Show that for $d = 2, 3, 6$ the number field K has class number $h_K = 1$ and for $d = 10$ it has class number $h_K = 2$.

Exercise 14. Show that for every number field K there exists a finite extension L/K such that every ideal of K becomes a principal ideal in L .

(Hint: You need to use that the class group is finite. For each class, try to find an element you need to adjoin to K such that the ideals in this class become principal).