Homework 3

Deadline: 5th December (23:55 JST), 2021

Exercise 8. Let $K \subset L \subset M$ be finite field extensions with $\mathrm{char}(K) = 0$ or $|K| < \infty$. Show that

$$\mathrm{Tr}_{L/K} \circ \mathrm{Tr}_{M/L} = \mathrm{Tr}_{M/K}$$
$$\mathrm{N}_{L/K} \circ \mathrm{N}_{M/L} = \mathrm{N}_{M/K}$$

Exercise 9. Let A be an integrally closed ring with field of fractions $K = \mathrm{Frac}(A)$. L/K a finite field extension and B is the integral closure of A in L. Show the following:

(i) $\beta \in L$ is integral over A if and only if $\min_K(\beta) \in A[X]$.

(ii) If $b \in B$ then $\mathrm{Tr}_{L/K}(b), \mathrm{N}_{L/K}(b) \in A$.

(iii) We have $b \in B^\times$ if and only if $\mathrm{N}_{L/K}(b) \in A^\times$.

Exercise 10. Let d_1, d_2 be two coprime square-free integers and $d_1 \equiv 1 \mod 4$. Determine the discriminant d_K of the number field $K = \mathbb{Q}(\sqrt{d_1}, \sqrt{d_2})$.

Exercise 11. Show that for any number field K the discriminant satisfies $d_K \equiv 0, 1 \mod 4$.

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